American Dialectic, Vol. 2, No. 2, 2012 (141-190)

Husserl's Genetic Philosophy of Arithmetic:

An Alternative Reading

Micah D. Tillman

In 1980, Dallas Willard argued that Husserl's first book, *Philosophy of Arithmetic* (*PA*),[1] contains two opposed understandings of calculation: one in the first twelve chapters, and the other in the thirteenth and final chapter. In later writings, from 1984, 1994, and 2003, it is clear that Willard continues to hold to this basic position.[2] Then, in 2011, Burt Hopkins offered much the same reading in *The Origin of the Logic of Symbolic Mathematics* (*Origin*).[3] Both Willard and Hopkins speak of Husserl beginning *PA*

[1] Edmund Husserl, *Philosophie der Arithmetik: Logische und psychologische Untersuchungen*, in Husserliana: Hua, vol. 12, *Philosophie der Arithmetic mit ergänzenden Texten* (1890–1901), ed. Lothar Eley (The Hague: Nijhoff, 1970); referred to hereafter as "*PA*." Page numbers refer to the Husserliana edition, unless preceded by "ET" in which case they refer to the English translation: *Philosophy of Arithmetic: Psychological and Logical Investigations*, in *Philosophy of Arithmetic: Psychological and Logical Investigations with Supplementary Texts from 1887–1901*, trans. Dallas Willard, *Edmund Husserl: Collected Works*, vol. 10, ed. Rudolf Bernet (Dordrecht: Kluwer, 2003). Referred to hereafter as "*PA(ET)*."

[2] Dallas Willard, "Husserl on a Logic that Failed," *The Philosophical Review* 89.1 (January 1980): 46–64; *Logic and the Objectivity of Knowledge: A Study in Husserl's Early Philosophy* (Athens, OH: Ohio University Press, 1984), 110–18 (referred to hereafter as "*LOK*"); translator's introduction to Edmund Husserl, *Early Writings in the Philosophy of Logic and Mathematics*, trans. Dallas Willard, *Edmund Husserl: Collected Works*, vol. 5, ed. Rudolf Bernet (Dordrecht: Kluwer, 1994), xiii–xiv (referred to hereafter as "*Early Writings*"); translator's introduction to *PA(ET)*, lvii, lviii–lix.

[3] Burt Hopkins, *The Origin of the Logic of Symbolic Mathematics: Edmund Husserl and Jacob Klein* (Bloomington, IN: Indiana University Press, 2011), 105–48. Willard sees the major change as occurring in the final chapter ("Logic that Failed," 62, 63), while Hopkins sees it as occurring in the final one or two chapters (see *Origin*, 112 for an emphasis on the final chapter, and 118 [n. 21], 123, and 136 for an emphasis on the final two chapters).

with one theory, "abandoning"[4] it as an error, and then ending *PA* with a second. Instead of going back and rewriting the book, Husserl apparently appended the new theory to the end, and published the whole, largely-mistaken, text.[5]

Hopkin's *Origin* shows that the image of *PA* as a text centered around two opposed accounts of calculation—one of which was a blunder—is alive and well after more than thirty years of Husserlian scholarship. But there is another way of reading *PA*—one that is at least suggested by Willard's own *Logic and the Objectivity of Knowledge*[6]—in which *PA* is seen as an exploration of the growth of calculation through three stages, moving from our common experience of numbers to the modern mathematician's arithmetical practice. This is the reading I propose to offer in what follows.[7]

My goal will be to charitably translate Husserl's account of arithmetical calculation into a simpler and more lucid form than that found in *PA*. *PA* is a poorly-edited text, marred by extensive side-trails, ambiguities of terminology, and arguments that, rather than being succinctly and coherently expressed, are begun in one place, dropped, and then picked up again in another. Those who have attempted any extended commentary on the book will know what a nightmare it can be. Yet, in spite of all this, my "translation"

4 See, for example, Willard, "Logic that Failed," 63; Hopkins, *Origin*, 106 (n. 2), 135 (n. 33), 511–12.

5 Willard holds that "the galley proofs for Part I of *Philosophie der Arithmetik* had been completed" (translator's introduction to *Early Writings*, xiii) when Husserl came to his realization that he had made a mistake.

6 Willard, *LOK*, 114–116 (see pp. 168-69, below). I suspect that the reading I will offer was inspired by this passage in Willard, though I was not conscious of the inspiration while writing it. Furthermore, as we will see below (pp. 168-69), my interpretation of the way in which the three stages of *PA*'s analysis are interconnected as a single whole differs from Willard's interpretation of the interrelation of the three stages.

7 I can make no claim to originality, however, for two reasons. First, there is the fact that I think the reading I will offer is the reading suggested by the text itself. Second, there is the fact that Willard also offers a three-stage reading of *PA* (see n. 6, above, and pp. 168-69, below).

will require only a touch of charity. There is a substantial and central vein of gold running through the rock of *PA* that deserves to be brought out—a vein I believe the Willard–Hopkins reading has left buried.

In the rest of this introduction, I will offer an overview of what I take to be Husserl's argument in *PA*. Then, in the first half of the paper, I will explore the progression of Husserl's argument in greater detail. Finally, in the second half of the paper, I will evaluate the analyses of Willard and Hopkins in light of the first half.

If we look at *PA* from a bird's-eye view, we will see that Husserl's investigation depends on three fundamental data: (1) our common, everyday experience of numbers in the real world, (2) arithmetical calculation as practiced by mathematicians like himself, and (3) the general assumption that arithmetic is about, or has to do with, the numbers we experience in the real world.[8] The ultimate task he sets himself—in spite of frequent debates with fellow scholars about other issues—is to justify the arithmetical practice of contemporary mathematicians in light of our everyday experience of numbers, and thereby to test the assumption that the arithmetic of modern mathematicians has to do with the numbers of common experience.

In the course of *Philosophy of Arithmetic*, Husserl shows how the current practice of arithmetical calculation develops naturally *and logically* out of our experience of numbers in the real world, and yet does not require the mathematician to intend those numbers consciously while calculating. Our everyday experience of numbers

[8] For example, Husserl writes, "After the discussion and resolution of the subtle questions connected with the analysis of the concepts *unity*, *multiplicity*, and *number*, our philosophical investigation proceeds to the task of making psychologically and logically intelligible the origination of a calculation-technique based upon those concepts, and of investigating the relationship of that technique to arithmetical science" (*PA*, 161 [ET 191]). Also, "*Arithmetic* is usually defined as the science of numbers" (ibid., 256 [ET 271]).

is the font of arithmetic as it is practiced by mathematicians, and thus arithmetic is grounded in that experience, but because that grounding is so secure, arithmetic need not be a conscious engagement with numbers. The general assumption that arithmetic is about numbers, therefore, is not strictly correct. To do arithmetic *as it is currently practiced* is not to actually deal with numbers but, rather, to mime or playact such an engagement in a logically-justifiable fashion.

To reiterate: Husserl is attempting, in *PA*, to see if we can get from our common experience of numbers to the modern practice of arithmetical calculation in a well-motivated and logically justifiable way. By the end of *PA*, we see that we can. Thus, rather than being the record of a failure and its last-minute correction, *PA* is a (labyrinthine) success.

I. Authentic Number Concepts and the First Type of Calculation

I.1 Authentic Number Concepts—The first stage in Husserl's argument is to show the kind of arithmetic that would naturally and logically grow out of our encounters with numbers themselves. Numbers, Husserl claims, are the various species or forms (in the Aristotelian sense) of groups considered merely as groups.[9] That is, when we take groups not as collections of these or those kinds of objects (for example, groups of humans, of cutlery, of physical

[9] "The number is the general form of multiplicity under which the totality of objects a, b, c falls" (ibid., 166 [ET 174]). "The numbers," Husserl says, "are the distinct species of the general concept of multiplicity" (ibid., 222 [ET 235]). Husserl refers to the various determinate types of multiplicity—i.e., to the particular numbers—as "concepts"; he says, for example, "the concept of multiplicity immediately splinters into a manifold of determinate concepts that are most sharply bounded off from each other: the *numbers*," and speaks of "concepts such as: *one and one; one, one and one; one, one, one and one*, and so forth" (ibid., 81 [ET 85]). (See also ibid., 181 [ET 191].)

entities, of mental entities), but merely as collections of objects *simpliciter* (of *whatever* type),[10] we find groups to fall into different species depending on *how many* members each group contains.[11] These species are what numbers are, and therefore these are the species we must seek to intuit if we wish to genuinely encounter numbers.

To intuit a number, we must first intuit groups as groups (of whatever types of objects), and to intuit a group as a group, we must intend its members, even though it is immaterial to us what types of members the group contains. Specifically, we must (a) distinctly intend each member in the group individually, and—simultaneously—(b) collectively intend all the members of the group together.[12] But it is not enough to intuit a single group in order to intuit a number. We must, in fact, experience two or more groups together *at the same time*.[13] This is because numbers are the forms or species of groups *qua* groups, and we can most easily make the form or species of a thing stand out for itself when we compare that thing with others.[14] For example, to intuit the species of shape we call "triangle," we either intuit multiple triangles (preferably of different colors and sizes) together, or—barring that—we intuit multiple shapes together, at least one of which is a triangle. It is in

[10] Ibid., 79 (ET 83).
[11] Ibid., 14 (ET 15), 130 (ET 137).
[12] Ibid., 74 (ET 77).
[13] "The multiplicity relations of equal, more and less essentially condition the origination of the number concepts. . . . [T]he determinate numbers, two, three, etc., presuppose a comparison and differentiation of delimited multiplicities, thought *in abstracto*, in terms of more and less. In order to be able to rise above the concept of the 'multiplicity of units' and form the series of numbers, two, three, etc., we must classify multiplicities of units. And this requires judgments of identity and non-identity. But an exact judgment of non-identity is in this case not possible without recognition of more or less" (ibid., 95 [ET 99–100]; compare 81–82 [ET 85–86]).
[14] Compare Robert Sokolowski, *Husserlian Meditations: How Words Present Things* (Evanston: Northwestern University Press, 1974), chapter 3, "How to Intuit an Essence."

the experience of comparing shaped objects or figures with each other that a particular species of shape can be brought to intuition for itself (and be distinguished from other things like mere color and pure extension).[15] Similarly, it is in the experience of comparing groups with each other that their species—in other words, numbers—can be brought to intuition for themselves.

The type of comparison we need in order to make a number stand out is *not* one in which we compare the types of members in one group with the types of members in another. Rather, as Husserl puts it, we must compare the groups in terms of "equal, more, and less."[16] In comparisons of intuitively given groups where it becomes clear that one is equal to, more than, or less than another, the number belonging to that group—the number of its members—becomes intuitable for itself. We can then conceive of this number as "this number here," on the basis of our intuition of that number. When we do this, we have the "authentic concept" (in Husserl's words) of that number. We are authentically conceiving of, or conceptualizing, the number, in that we are conceiving of it as it actually is, and as it is actually given.[17]

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[15] I take this to be a common account of how one can intuitively identify, or distinguish between, abstracta like color, shape, and extension, though it is not one to which Husserl appeals in *PA*.

[16] Husserl, *PA*, 95 (ET 99). For Husserl's discussion of exactly what goes into comparing two groups in terms of equal, more, and less, see the whole of chapter 5 of *PA*, especially 91–93 (ET 96–97), and 93–94 (ET 98–99).

[17] See, for example, ibid., 193 (ET 205). Willard writes that "authentic representations" are "intuitive acts in which small multiplicities or sets, and the corresponding small numbers, present themselves exactly as what they are, and are 'authentically' grasped" (*LOK*, 114). He furthermore holds that "representation" (*Vorstellung*, for which I would prefer "presentation") and "concept" (*Begriff*) are equivalent "without interesting exception" in Husserl's "earliest writings" (ibid., 26). "A concept or representation [i.e., presentation] is treated by Husserl as a repeatable and shareable *thought*" (ibid., 27).

In explaining here what it means to have the authentic concept of a number—that is, the way in which we would conceive of a number authentically—I have had to help Husserl's exposition along, pulling together widely-scattered claims about authentic concepts and presentations and then trying to formulate

Unfortunately for humans, however, authentic number concepts are only available to us for small numbers. It takes so much mental power to be able to carry off the intuitive group comparisons necessary for intuiting a number itself, that we can only manage intuitions of numbers up to about a dozen.[18] There is nothing logically-necessary about this limitation, however. It is merely empirical. Beings with more powerful minds could conceivably intuit much higher numbers, and thus authentically conceive of much higher numbers.[19]

I.2 The First Type of Calculation—Now, Husserl asks, what kind of calculation would grow out of our intuitive experience of numbers alone? His answer is: a kind of calculation that is very limited, and, in fact, only capable of two operations: uniting two intuited numbers into a larger number (by conjoining the groups to which they belong), or splitting a number into smaller numbers (by splitting the group to which it belongs into smaller groups). The first operation would be what we call "addition." The second operation would be either what we call "subtraction," or what we call "division," depending on how it is carried out.[20]

The problem is that this way of calculating only partially matches the way arithmetic is currently practiced by

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Husserl's basic position in language that would make sense in our current idiom. Nowhere, for example, does Husserl say that to have the authentic concept of something, one must verbally articulate the way in which one is conceiving of the thing (for example, "this number here"). However, by translating the authentic concept of a number into linguistic form, I hope to make the way in which we are conceiving of that number (when we are conceiving of it authentically) clearer. (Compare the way in which Husserl speaks of an "inauthentic" or "symbolic concept": "In what way do we have, for example, the concept of a twenty-place number? We obviously first think the mere concept: a certain number which corresponds to *this* sign complex" [Husserl, *PA*, 242 {ET 256}].) Nevertheless, what matters when we have an authentic concept of a number is the way in which we are conceiving of the number, not the words we use (if any at all).

[18] Ibid., 192 (ET 202). Compare 196–97 (ET 209).

[19] Ibid., 191–92 (ET 202).

[20] Ibid., 182–90 (ET 192–200).

mathematicians.[21] It is a *legitimate* way of calculating, and is the one closest to our intuitive experience of numbers. However, it simply does not have enough overlap with modern arithmetical practice in order to connect that practice to our common experience of numbers (and thereby to justify that practice). Thus, the discovery that this is the type of calculation we would develop based only on our *intuitive* experience of numbers shows us that there must be a part of our experience of numbers that we have missed. Husserl then proceeds to add that other part to his account, and to describe the second type of calculation that would then develop.

II. Inauthentic Number Concepts and the Second Type of Calculation

II.1 Inauthentic Number Concepts—The fact that we can only authentically conceive of the cardinal numbers up to about twelve does *not* mean that we cannot intend numbers larger than twelve. We do this all the time. The best we can manage in such cases, however, is not an authentic concept, but rather what Husserl calls an "inauthentic" or "symbolic" concept. Instead of conceiving of the number as "this number here," on the basis of intuiting the number in question, we merely conceive of it as, "whatever number of members is in this group here." We know, since we are perceiving a group, that there must be *some* number of members before us, but we cannot bring that number to intuitive givenness for itself.[22]

Husserl calls this way of conceptualizing numbers "symbolic" because in it we are employing a concept that stands in for, and is logically equivalent to, the authentic concept.[23] The notion of one

[21] Ibid., 190–92 (ET 200–3).
[22] Ibid., 222–23 (ET 235–36).
[23] Ibid., 193–94 (ET 205–7).

concept being a symbol or sign for another, however, is not one I find helpful. Instead, it might be clearer to say that in symbolic concepts, rather than conceiving of the number as we intuit it, (a) we are using something other than the number to stand in for it, or (b) we are intending the number *by way of* something else. In the case where we conceive of the number as "whatever number of members is in this group here," it is the group itself that stands in for its number, or is that-by-way-of-which we intend its number. However, such a way of conceiving of numbers is exceedingly vague, Husserl notes.[24] Therefore, in the human attempt to deal with numbers beyond those we can actually intuit, the need arises to find a way to make our symbolic ("inauthentic") concepts more distinct.

II.2 Making Our Symbolic Concepts More Authentic—This is accomplished, Husserl argues, by conceiving of large groups—whose associated numbers are too large to intuit—as collections of smaller groups—whose associated numbers are *not* too large to intuit.[25] Thus, instead of conceiving of a number as "whatever number of members is in this group," we might think of the group as divided into three smaller groups, each of whose numbers we could intuit. Upon finding that one subgroup has eight members, the next has six, and the last has nine, we can then conceive of the original number as "whatever number is eight plus six plus nine."[26] In doing this, we have introduced at least a modicum of "authenticity" into our inauthentic (symbolic) way of conceptualizing the number in question. Our concept is still symbolic; we are still conceiving of the

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[24] Ibid., 223 (ET 236).
[25] Ibid., *PA*, 224 (ET 236–37).
[26] Compare Willard, translator's introduction to *PA(ET)*, lv: "Undoubtedly the most characteristic expressions of simple arithmetic are of the following types: '85 + 36,' '92 – 13,' '7 · 37,' '92 ÷ 4,' and so forth, and complications thereof. These may be read as descriptive phrases—as, for example, 'the sum of 85 and 36,' or 'the difference between 92 and 13.' . . . They are clearly distinguished, sense perceptible signs that refer to one and only one of the cardinal numbers."

number not as it is actually given to us (since it is not, and cannot be, actually given to us), but by way of a sum of smaller numbers. That sum of smaller numbers is, as it were, standing in for the number itself.[27] But since those smaller numbers are intuitable, this way of conceiving of the whole number is closer to being authentic than if we were simply to conceive of the number as "whatever number of members is in this group here."

We can make our symbolic concept of a large number more authentic (though not fully authentic) by conceiving of the group to which it belongs as a more or less arbitrary collection of smaller groups, each of whose numbers we could intuit. It would be more helpful, however, Husserl argues, if we systematized this practice, rather than going about it arbitrarily.[28] The modern world, for instance, eventually settled on the method—to put it roughly—of conceiving of large groups as collections of groups of *ten*. That is, they adopted what we now call the "base-ten" number system. Thus, we conceive of the United States of America as five groups of ten states, and say the number of states is "whatever number is five tens" (which is eventually shortened to "fif-ty"). We conceive of the number of keys on a piano as eight groups of ten keys, with eight keys left over, and say the number of keys is "whatever number is eight tens, and eight" (which is eventually shortened to "eight-ty-eight").

Furthermore, if we are dealing with a very large group, we even group the *groups of ten* into as many groups of ten as possible. The days in a year, for instance, can be conceived of as three groups

[27] Compare Husserl, *PA*, 223 (ET 236): "[F]or the purposes of enumerating and calculating . . . we require symbolic formations . . . which, coordinated in their rigorous distinctiveness with the true—but to us inaccessible—number concepts 'in themselves,' are well suited to stand in for those concepts." Here we see, once again, Husserl's unfortunate tendency to refer to numbers as concepts in *PA* (see n. 9, above).

[28] Ibid., 225–26 (ET 238–39), 235–37 (ET 249–51).

of ten groups of ten days, and six groups of ten days, with five days left over. We can then say that the number of days in a year is "whatever number is three sets of ten tens, six tens, and five." Or, introducing the word "hundred" for ten tens, we can shorten this, in English, to "three-hundred six-ty five." And so on for larger numbers.[29]

Husserl proposes, therefore, that the point of the normal base-ten number system is that it provides an algorithmic way of conceiving of the large numbers we can only conceptualize symbolically or inauthentically, such that our concepts become more authentic and distinct. This makes us better able to work with these large numbers mathematically. The base-ten system, however, is not the only one. Husserl suggests that the base-twelve system might in fact be more useful.[30] However, *any* such algorithmic system for articulating large numbers produces what Husserl calls "systematic numbers,"[31] and this is key to what follows.

II.3 Systematic and Nonsystematic Number Concepts—The "systematic numbers" are not actually numbers. They are, rather a special class of symbolic number *concepts*—ways of conceiving of numbers as collections of smaller numbers. For example, in the base-ten system, "whatever number is three tens, and two" is a systematic number; that is, it is a way of conceiving of a number that is consistent with the rules of the base-ten system. However, the equivalent concept, "whatever number is two twelves, and eight" is a nonsystematic number; that is, it is a way of conceiving of a number that is *not* consistent with the rules of the base-ten system. It presents the number as a collection of twelves, with a few left over, rather than as a collection of tens, with a few left over. Note,

[29] On all of this, see ibid., 228–33 (ET 241–47).
[30] Ibid., 237 (ET 250–51).
[31] Ibid., 233 (ET 247).

however, that "whatever number is three tens, and two," and "whatever number is two twelves, and eight," are ways of conceiving of the *same* number (the number we would call "thirty-two"). It is just that one fits the base-ten system, and thus is "systematic," while the other does not fit the system, and thus is "nonsystematic."[32]

Once we have adopted any particular number system (base-ten, base-two, base-twelve, etcetera) our task will then be to translate any "nonsystematic" number concepts we encounter into their equivalent systematic concepts.[33] That is, given any way of conceiving of a number which does not match the algorithmic system we have adopted, our job will be to discover which way of conceiving of that *same* number *would* fit into our system. For example, let us assume we have adopted the standard base-ten system. If we are then confronted with the symbolic concept, "whatever number is three sixes, and nine," our job is to figure out how to conceptualize that same number (whatever it is) in a way that matches our system. Doing this—discovering the systematic way of conceiving of the same number that we are at first conceptualizing in a nonsystematic way—is a type of calculation. After a little work, for example, we discover that "whatever number is three sixes and nine" is the same as "whatever number is two tens and seven."

II.4 The Second Type of Calculation—But how do we do this? A proper number system provides us with an algorithm for translating each nonsystematic way of conceptualizing a number into a systematic way of conceptualizing the same number.[34] For

[32] Ibid., 260–61 (ET 275–77).
[33] Ibid., 260–61 (ET 275–77).
[34] See, for example, ibid., 257 (272), 259–72 (ET 274–88). Notice that in 259–72 (ET 274–88), Husserl is primarily discussing conceptual calculation, while making side-comments about the equivalence of the third type of calculation, which proceeds by using signs alone. Most of chapter 13 of *PA*, in other words, is devoted to describing conceptual calculation, even though Husserl announces at the beginning of chapter 13 that he only wishes to deal with sign-only calculation.

example, when confronted with "whatever number is three sixes, and nine," the algorithm might tell us to translate the "three sixes" into a systematic form first. "Three sixes" is the same as "six and six and six." "Six and six" is "ten with two left over." "Ten with two left over and six" is "ten with eight left over." So, "three sixes" is "ten and eight." That means, "whatever number is three sixes, and nine" is equivalent to "whatever number is ten and eight and nine." The algorithm might then tell us to translate the "eight and nine." "Eight and nine" is "ten with seven left over." So, "whatever number is ten and eight and nine" is the same as, "whatever number is ten and ten and seven." Finally, the algorithm would tell us to translate the "ten and ten" into "two tens," and thus we would end up with "whatever number is two tens, and seven." This would be the process of calculating "whatever number is three sixes and nine" in the base-ten system.

Now, this description of calculation certainly seems closer to the practice of mathematics than our first attempt, when we had taken into account only those numbers we could conceptualize authentically. We are, therefore, making progress toward connecting our common experience of numbers in the real world with the way in which mathematicians practice arithmetic. What we have discovered is that the way mathematicians actually calculate makes more sense if we see them as (a) thinking not only of those numbers we can intuit, but also (b) conceiving of those numbers we cannot intuit as collections of smaller numbers that we can intuit, and then (c) trying to algorithmically translate those ways of conceiving of numbers which do not fit a given system into ways of conceiving of numbers that do fit the system.

The reason he offers for his focus on conceptual calculation is that an understanding of conceptual calculation is required to show the validity of sign-only calculation (259 [ET 274]). (See pp. 164-66, below, for further discussion.)

But do mathematicians really think their calculations through, moving algorithmically from a nonsystematic way of conceptualizing an unintuitable number to a systematic way of conceptualizing the same number? Husserl's answer is, "No."[35] There is an important link in the chain connecting our common experience of numbers with the mathematician's practice which we have not yet included. Just as we had to add those numbers we could only conceptualize inauthentically to those numbers we can conceptualize authentically, we now must add number signs.

III. Number Signs and the Third Type of Calculation

III.1 The Sign System—A proper number system includes not only an algorithm for conceiving of large numbers as collections of smaller numbers, and an algorithm for translating nonsystematic ways of conceiving of numbers into systematic ways of conceiving of numbers, but also a system of *signs* to assist in these translations. The sign system provided by a proper number system will include: (a) a unique sign for each of the basic numbers zero up to (but not including) the base number of the system, (b) an algorithm for composing the signs for larger numbers out of the signs for the basic numbers, which mirrors the algorithm for conceiving of large numbers as collections of smaller numbers,[36] and (c) an algorithm for translating the signs representing numbers as conceived of in nonsystematic ways into the signs for the same numbers as conceived in systematic ways.[37] Furthermore, a set of operation signs will also have to be included in order to properly represent

[35] Husserl, *PA*, 257–59 (ET 272–74).
[36] Ibid., 228 (ET 241–42), 237 (ET 251).
[37] See ibid., 257–58 (ET 272–73), 264–65 (ET 280), 266–67 (ET 282), 268 (ET 284) (and 270–71 [ET 286–87]), in light of 228 (ET 241–42), and 237–39 (ET 251–53).

numbers as being conceived of in nonsystematic ways. (For example, "whatever number is three less than five sevens" would need to be represented as "5 × 7 – 3," while its equivalent systematic concept, "whatever number is three tens, and two" can be represented with numerals alone: "32.")

With a proper number system, then, we would not have to calculate only by conceiving of a number in a nonsystematic way, and then following the rules for translating that way of conceiving of the number into a systematic way of conceiving of the same number. We could also employ the signs, and algorithms for constructing and transforming them, which are provided by the system, to assist us in our conceptualizations and translations. And once we introduce the sign part of the number system, further advances become possible.

First, we become able to deal with numbers of which we cannot conceive inauthentically with any sort of distinctness.[38] Take, for example, the number of people in China. According to the World Bank, China's population in 2010 was 1,338,299,512.[39] If we were to try to fully conceive of the number that corresponds with that sign, we would have to be able to understandingly grasp the following: "whatever number is *one* set of ten sets of ten sets of ten sets of ten sets of ten sets of ten sets of ten sets of ten tens, *three* sets of ten sets of ten sets of ten sets of ten sets of ten sets of ten sets of ten tens, *three* sets of ten sets of ten sets of ten sets of ten sets of ten sets of ten tens, *eight* sets of ten sets of ten sets of ten sets of ten sets of ten tens, *two* sets of ten sets of ten sets of ten sets of ten tens, *nine* sets of ten sets of ten sets of ten tens, *nine* sets of ten sets of ten tens, *five* sets of ten tens, *one* ten, and *two*." And it is simply impossible for a human to actually think this. We can say the words, but we are

[38] Ibid., 240–42 (ET 254–56).
[39] The World Bank, "Population, total | Data | Table," n. d., http://data.worldbank.org/indicator/SP.POP.TOTL (accessed 21 February, 2012).

not actually conceiving of the number in question (in anything but the vaguest of ways) as we make the noises. The best we can do is to conceive of the number as, "whatever number is represented by '1,338,299,512'."[40]

What is wonderful about the number system we use, however, is that conceiving of the number of people in China as "whatever number is represented by '1,338,299,512'" is good enough for arithmetical purposes. It is no more precise in our minds than "whatever number of members is in this group here," and yet the systematic way in which the numeral has been constructed on the page—and the algorithmic system of transformations to which it belongs—allows us to calculate with it. The algorithm for constructing complex number signs out of simple number signs so exactly mirrors the algorithm for conceiving of large numbers as collections of smaller numbers that the number signs allow us to tap into, or link up with, the system of number conceptualization, even when we cannot distinctly conceive of the numbers with which we are trying to work.

III.2 The Third Type of Calculation—In fact, the number system we have is so good that we do not even have to conceive of numbers via the number signs at all. We can simply work with the number signs themselves, and forget all about the numbers.[41] The signs are like the control levers for the system, and once we grasp them, we can make the whole machinery of the system function, even when we no longer have any mental access to all the conceptual transformations going on inside. Using the signs, we go through the outward motions of the conceptual transformations, and yet do not

[40] Husserl, *PA*, 242 (ET 256): "In what way do we have, for example, the concept of a twenty-place number? We obviously first think the mere concept: a certain number which corresponds to *this* sign complex."
[41] Ibid., 257–58 (ET 272–73).

have to perform those transformations ourselves.[42] For example, if we encounter "1234 + 567" we do not have to think, "whatever number is the number represented by 1234 plus the number represented by 567," and then think, "whatever number is one set of ten sets of ten tens, two sets of ten tens, three tens, and four, plus five sets of ten tens, six tens, and seven," in order to calculate. Instead, we can "mechanically" follow the algorithm for finding the complex sign equivalent to "1234 + 567" (which happens to be "1801"). Husserl refers to this as "mechanical" calculation.[43]

How does the algorithm for mechanical calculation work? In this case, as follows. First, the algorithm tells us to add the numerals in the "ones place." These are "4" and "7." Thus, we implicitly have a new sign complex: "4 + 7." Now, given the fact that we have memorized our addition tables,[44] we know that the sign complex "4 + 7" can be replaced by the complex numeral "11." We now write the numeral in this new complex numeral's "ones place" as the numeral for the "ones place" in our answer (". . . 1") and add the "1" in the "tens place" to the "3" and "6" that are in the "tens places" of "1234" and "567." Thus, we implicitly construct a new sign complex: "1 + 3 + 6." Once again, because we have memorized our basic addition tables, we know that "1 + 3" can be replaced by "4," and "4 + 6" can be replaced by "10." We then take the numeral in the "ones place" of "10," write it as the numeral in the "tens place" of our answer (". . .

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[42] Ibid., 259 (ET 274): "Only the systematic combination of the concepts and their interrelationships, which underlie the calculation, can account for the fact that the corresponding *designations* interlock to form a coherently developed system, and that thereby we have certainty that to any derivation of signs and sign-relations from given ones, which is valid in the sense prescribed by the rules for the *symbolism*, there must correspond a derivation of the concepts and conceptual relations from *concepts* given, valid in the sense that *thoughts* are. Accordingly, for the grounding of the *calculational methods in arithmetic* we will also have to go back to the *number concepts* and to their *forms of combination*."

[43] See, for example, ibid., 238–39 (ET 252–53), 250 (ET 265), 267 (ET 283–83).268 (ET 284), etcetera.

[44] On addition tables, see ibid., 266–67 (ET 282).

01"), and add the "1" in the "tens place" to the "2" and "5" that are in the "hundreds place" of "1234" and "567." Thus, we implicitly produce another new sign complex: "1 + 2 + 5." Once again, because we have memorized our basic addition tables, we know that "1 + 2" can be replaced by "3," and "3 + 5" can be replaced by "8." We then write this "8" in the "hundreds place" of our answer (". . . 801"). And finally, we add the numerals in the "thousands place" in "1234" and "567." Since there is no numeral in the "thousands place" in "567," we use a "0" instead, and implicitly produce: "1 + 0." And, since we have memorized our basic addition tables, we know that any numeral "+ 0" can be replaced by that numeral. So, we write a "1" in the thousands place of our answer, and produce: "1801."[45]

In none of this did we have to have numbers in mind. All we had to do was memorize a set of basic sign equivalences, and a certain methodical process for producing appropriate sign complexes and recording results. In other words, because of the sign system that was designed to help us conceptually calculate in a given number system (the second type of calculation, discussed above), it is possible to calculate without conceiving of numbers in any way. And this, in fact, is how mathematicians work, Husserl argues.[46] It is not necessary that they practice mathematics in this way, of course. They could (logically speaking) calculate in either of the two ways we have discussed above. But this way of calculating is much more powerful, given our contingent human limitations. It allows mathematicians to accomplish much more than they could

[45] On all of this, see ibid., 265–67 (ET 281–82), which describes the method in terms of conceptual calculation, and then points out that the same can be accomplished with symbols alone. See also, ibid., 238 (ET 252).

[46] See ibid., 257–58 (ET 272–73), 259 (ET 274). "*Most* of what goes on in the practice of the arithmetician is not at all a matter of representing or conceiving of numbers and number relations, although that obviously remains the *ultimate* goal. Rather it is a matter of working with one or more algorithms" (Willard, *LOK*, 116).

with either of the two previous ways of calculating. Thus, the actual practice of mathematicians is one of sign manipulation, not number conceptualization (whether symbolic, as in the second type of calculation, or authentic, as in the first type).

IV. Review

We have at last traveled the full length of the chain connecting our common experience of numbers to the mathematician's practice of calculation in the 19th century (up through today). In extra- or premathematical life, we experience numbers either as intuitively given, or as intended but not given. That is, we conceive of them authentically or symbolically. The need to make our symbolic concepts more authentic and distinct leads us to conceive of numbers as collections of smaller, genuinely intuitable numbers. The need to organize and work with these ways of conceiving of numbers leads us to introduce number systems. The number systems provide us with algorithms for systematically conceiving of numbers, and translating extrasystematic ways of conceiving into systematic ones. To help in both processes, the number system also provides a system of signs that mirrors the system of conceptualizations and translations. This system, grounded in, and derived from, the number system to which it belongs, can then be used to calculate, even if we no longer try to conceive of the numbers the signs were originally designed to help us articulate.

Each step in this process arises out of the needs created by the previous step, and represents not only a helpful solution to those needs, but also a logically justifiable solution.[47] Thus, even though

[47] Compare ibid., 233–34 (ET 247): "[W]e find the view objectionable according to which the number system is a mere tool to provide a systematic nomenclature for the natural numbers, intending economy of symbolism. . . . Only a tiny opening segment of that sequence [of natural numbers] is given to us. Certainly we can

the modern mathematician may not concern herself with numbers while she is carrying out her calculations, her practice is connected by a chain of valid developments back to our common experience of numbers in the real world.

Husserl shows in *PA*, therefore, how three different calculational methods arise in the process of the development of mathematics out of our common experience of numbers in the real world. The first is calculating with intuitively given numbers. The second is calculating with numbers we cannot intuit, and yet have conceived of as collections of numbers we can intuit. The third is calculating with the number signs that were designed to represent the numbers we cannot intuit as being collections of numbers we can intuit. All three are legitimate ways of calculating, but only the third matches calculation as it is currently practiced by mathematicians. And since Husserl wishes to understand and justify the actual practice of contemporary mathematicians, he achieves his goal by showing how the third type of calculation arises naturally and logically out of the previous two.

Husserl's philosophy of mathematics, in other words, is a genetic one in which we see how mathematics develops. Later stages

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conceptualize the Idea of an unlimited continuation of it, but the actual continuation, even for only the moderate range involved in the ordinary practice of calculating, already places demands upon our mental capabilities which we cannot fulfill. The impossibility of being able to solve more demanding problems of calculation in such a primitive way was the source for those logical postulates whose satisfaction led to a new and farther reaching method of concept formation. And so the number-systematic arrived at (specifically, our ordinary decimal system) is not then, a mere method of symbolizing concepts which are given, but rather one of constructing new concepts and simultaneously designating them along with their construction."

Also, compare ibid., 242 (ET 256): "The concepts are of course indispensable for anyone who for the first time grasps the essence and aim of the number system, or who at some later time has the need to bring to consciousness the full conceptual content of a complex number sign. Reflections on the concepts are the sources out of which arise the rules of all arithmetical operating. But the mere sensible signs continually underlie practical activities."

in the evolution of mathematics, and thus in Husserl's account, "leave previous stages behind" not by writing them off as mistakes, but by responding to the needs created by them. *PA* is, contra Willard and Hopkins, not an exposition of a legitimate theory of calculation appended to a much longer exposition of an erroneous one, but a detailed exploration of the continuous, organic, and logical growth of the sophisticated practice of today's professional mathematicians out of the common experience of numbers.

With this general understanding in hand, we may now turn to a more detailed study of, and response to, the Willard–Hopkins reading of *PA*. We will take the relevant texts in chronological order, beginning with Willard's "Husserl on a Logic that Failed."

V. Response to Willard

V.1 "Husserl on a Logic that Failed" (1980)—Husserl began his philosophical investigation of arithmetic from Weierstrass's assumption "that arithmetic was grounded in the concept of number alone," according to Willard.[48] However, in his exploration of number systems (*PA*, chapter 12), Husserl discovered that "*some* system of symbols, including of course the rules that govern their formation and transformation, [is] prior in the order of knowledge to 'that never-ending series of concepts, which mathematicians call "positive whole numbers"' (*PA*, 294)."[49] Willard expresses the discovery as follows:

> [Husserl] holds that our minds can represent numbers of even a very modest size only through their structural correlation with a sensible symbol that shows, in its composition, its place in an ordered series of sensible symbols (the decadal numerals, for example) running parallel with the number series. Hence, the *logical* content of thoughts of most numbers—that is, what is *meant* in them—essentially involves a reference

[48] Willard, "Logic that Failed," 53; see also, 60, 62.
[49] Ibid., 61.

> to some system of symbols. That is, the object (a number) to which the concept of number applies is, in most cases, something present to the mind only in virtue of an analogy of its structure to the structure of a symbol. For example, the number 3,786 is represented by means of its analogy of structure to the numeral "3,786"—or, for Augustine, "MMMDCCLXXXVI."[50]

The first four chapters of *PA* derive from Husserl's 1887 *Habilitationsschrift*,[51] while the realization just described is not expressed until chapter 12, which was written later. As Husserl progressed through the writing of *PA*, in other words, he ends up "reject[ing] the dependence of calculation upon number"—the dependence upon which he based *PA*'s opening chapters—"and, instead, defines calculation in terms of symbolic technique."[52]

In support of this reading, Willard quotes a letter Husserl wrote to Carl Stumpf as follows:

> The opinion by which I [Husserl] was still guided in the elaboration of my *Habilitationsschrift* [in other words, at least the material in "Part One" of the *Philosophy of Arithmetic*] to the effect that the concept of cardinal number forms the foundation of general arithmetic, soon proved to be false. (I was already led to this in the analysis of the ordinal number.) By no clever devices, by no "inauthentic representing," can one derive negative, rational, irrational, and the various sorts of complex numbers from the concept of the cardinal number. The same is true of the concept of the ordinal, of the concept of magnitude, and so on. And these concepts themselves are not logical specifications of the cardinal concept.[53]

Here, it seems clear that Husserl changed his mind between finishing his *Habilitationsschrift* and finishing *PA*, even though he incorporated his *Habilitationsschrift* into *PA*. When he finished his

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[50] Ibid.

[51] The German of Husserl's *Habilitationsschrift*, from 1887, can be found in Hua XII, 289–338, while Willard's English translation can be found in *PA(ET)*, 305–356. In Hua XII, Eley labels the *Habilitationsschrift* the "Ursprünglische Fassung des Textes [*PA*] bis Kapitel IV" (289), which Willard translates as the "Original Version of the Text [*PA*] through Chapter IV" (*PA(ET)*, 306).

[52] Willard, "Logic that Failed," 62.

[53] Ibid., 63; Willard's interpolation. Willard later translated the entire letter as "Letter from E. Husserl to Carl Stumpf," in Edmund Husserl, *Early Writings*, 12–19.

Habilitationsschrift, he thought that "general arithmetic" was based on "the concept of cardinal number." Sometime later, he realized that this was "false" because, for instance, "negative, rational, irrational, and the various sorts of complex numbers" do not arise by somehow inauthentically conceiving of cardinal numbers. Rather, since arithmetic actually is a "technique of signs,"[54] such numbers must somehow arise out of the activity of calculative sign-manipulation.

Husserl's admission of a change of mind is not so obvious within *PA*, however. After saying that calculation is "*any symbolic derivation of numbers from numbers which is substantially based on rule-governed operations with sense perceptible signs*"[55] Husserl writes that "[t]he relationship between arithmetic and calculational technique, with this new concept of calculating . . . has certainly changed."[56] Willard notes:

> With these two words ["certainly changed"] we have the whole of Husserl's acknowledgement of the momentous fact that the conceptualization which had guided the entire enterprise, whose development is expressed in the *Philosophy of Arithmetic,* is in fact abandoned in the book's final chapter.[57]

And this seems to be bolstered by Husserl's claim that the new concept of calculation is "from now on . . . the only one we wish to use."[58] Husserl will have no more of the old understanding of arithmetic, it seems, which took calculation to be an activity with number concepts. For the rest of the chapter (and hence, for the rest of the book), he will apparently stick to the new understanding of arithmetic, which understands calculation as an activity with signs alone.

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[54] Willard, "Logic that Failed," 64, quoting Husserl's letter to Stumpf.
[55] Husserl, *PA*, 258 (ET 272–73).
[56] Ibid., 259 (ET 274).
[57] Willard, "Logic that Failed," 63.
[58] Husserl, *PA*, 259 (ET 274).

Nevertheless, I would note, *in the very next paragraph*, Husserl writes this:

> If a discipline at the height of its development needs no other than calculational means for the solution of its problems, it still certainly cannot dispense with that which is conceptual in its beginnings, where it is a matter of providing a logical foundation for calculation. Only the systematic combination of the concepts and their interrelationships, which underlie the calculation, can account for the fact that the corresponding *designations* interlock to form a coherently developed system, and that thereby we have certainty that to any derivation of signs and sign-relations from given ones, which is valid in the sense prescribed by the rules for the *symbolism*, there must correspond a derivation of concepts and conceptual relations from *concepts* given, valid in the sense that *thoughts* are. Accordingly, for the grounding of the *calculational methods in arithmetic* we will also have to go back to the *number concepts* and to their *forms of combination*.[59]

Husserl then spends the rest of the chapter, and thus the rest of the book, describing how one would calculate with number concepts, making occasional side-comments about how such calculations could be translated into mere sign manipulation. That is, he spends the rest of the chapter and book focused *not* on calculation-as-sign-manipulation, but (a) on the second type of calculation we discussed above, and (b) the way in which that type of calculation can be reenacted (as it were) in calculation-as-sign-manipulation. Evidently, he believes very firmly that mechanical calculation can only be fully understood, and fully legitimated, if we see how it is grounded in the second type of calculation (calculation with inauthentic number concepts).

Thus, when Willard writes the following, it seems to me that he is largely correct, though his final conclusion is misleading.

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[59] Ibid. See also ibid., 242 (ET 256): "The concepts are of course indispensable for anyone who for the first time grasps the essence and aim of the number system, or who at some later time has the need to bring to consciousness the full conceptual content of a complex number sign. Reflections on the concepts are the sources out of which arise the rules of all arithmetical operating."

> [I]n order to construct the (necessarily inauthentic) *representation of* 2,345, for example, I must use the system of decadal numerals. And in order to *know that* 2,345 + 257,892 = 260,237, I must follow certain rules built upon that numeral system, but also incorporating the most abstract principles of valid thought. But the numeral system itself, with the associated formal laws, requires a philosophical clarification of its own. It (and not merely a fundamental concept or set of concepts) provides the theoretical unity of my cognitive experiences of the series of whole numbers and, through appropriate abstractions and extensions, of the entire domain of pure mathematics. Also, it is presupposed in, and cannot be explained by, symbolic (or inauthentic) representing and knowing within the domain of mathematics.[60]

Here, Willard is right to note our reliance upon number signs, and sign systems, not only in intending the number two-thousand, three-hundred, forty-five, but also calculating its summation with other large numbers. His point that numeral systems are not transparently intelligible, and thus are in need of philosophical explication, is likewise extremely important. Furthermore, I find his claim that it is our numeral systems—"not merely a fundamental concept or set of concepts"—which allow us to coherently engage in mathematical practice, to be very well put. This fact seems to be, based on the evidence offered by Willard, something that Husserl came to realize between finishing his *Habilitationsschrift* and finishing *PA*. However, to say that the numeral system is "presupposed in, and cannot be explained by, symbolic (or inauthentic) representing and knowing within the domain of mathematics" seems to me to conflict with Husserl's own explicit claim—which comes *after* his purported abandonment of his older, mistaken understanding of calculation—that

> [o]nly the systematic combination of the concepts and their interrelationships, which underlie the calculation, can account for the fact that the corresponding *designations* interlock to form a coherently developed system, and that thereby we have certainty that to any derivation of signs and sign-relations from given ones, which is valid in the sense prescribed by the rules for the *symbolism*, there must

[60] Willard, "Logic that Failed," 64.

> correspond a derivation of concepts and conceptual relations from *concepts* given, valid in the sense that *thoughts* are. Accordingly, for the grounding of the *calculational methods in arithmetic* we will also have to go back to the *number concepts* and to their *forms of combination*.[61]

Thus, if all Willard means by the numeral system's being "presupposed in symbolic (or inauthentic) representing and knowing within the domain of mathematics" is that we find such systems helpful in working with the *number* systems to which they are attached, then I can agree. However, I cannot reconcile the claim that numeral systems "cannot be explained by, symbolic (or inauthentic) representing and knowing within the domain of mathematics" with Husserl's own claim—which comes *immediately after* Husserl purportedly admits to abandoning his initial, mistaken view—that the only way to properly understand and validate mechanical calculation (which does not involve representing numbers, but rather works with numerals alone) is to understand conceptual calculation (which does involve representing numbers, albeit in an inauthentic way).

Thus, I would argue that Willard's conclusion in "Husserl on a Logic that Failed"—that Husserl ultimately rejects his understanding of calculation as being based upon number concepts—is misleading. What Husserl in fact discovers is that there are three natural, or well-motivated types of calculation. The first two directly involve working with number concepts (in other words, they directly involve conceptualizing numbers), with the second growing out of the first. The third adopts the sign system developed to facilitate the second type of calculation, and works with that system alone, without conceptualizing numbers. Nevertheless, it is derived from, and justified by, the system of number concepts developed for the second type of calculation. The third type of

[61] Husserl, *PA*, 259 (ET 274).

calculation is not a direct involvement with number concepts, and yet it remains fundamentally based upon the number concepts. Willard is correct, therefore, that Husserl discovers that mathematics—as it is actually practiced by contemporary mathematicians—"is [not] based entirely upon the concept of number."[62] It is, rather, based upon the concept of number *by way of* a sign system which is itself based upon the concept of number.

What Husserl does, then, between writing his *Habilitationsschrift* and completing *PA*, amounts to discovering not that his original theory of arithmetic was wrong, but rather that an important new layer had to be included in his theory's fully-developed version. This is why *PA* hangs together as a work; the final chapter is not an about-face, but the logical next—and concluding—step in Husserl's argument.

Furthermore, we must be careful not to confuse the use of signs in type two calculation, with the use of signs in type three calculation. The numeral system is first developed to facilitate our conceptualization of numbers, and thus to facilitate our type two calculations. Thus, the signs enter into "the *logical* content of thoughts of most numbers," as Willard says.[63] In fact, the signs are so helpful to us in expanding our ability to deal with and conceptualize numbers that it may even be right to say with Willard that "*some* system of symbols, including of course the rules that govern their formation and transformation, [is] prior in the order of knowledge to 'that never-ending series of concepts, which mathematicians call "positive whole numbers"' (PA. 294)."[64] But this does not mean that the sign system is prior to the number system, nor that type three (mechanical) calculation is prior to type two

[62] Willard, "Logic that Failed," 62.
[63] Ibid., 61.
[64] Ibid.

calculation. Calculation using numerals alone (type three) is not prior to calculation using numerals to facilitate our conceptualizations of numbers (which is one way of engaging in type two calculation). Any system of symbols is itself only developed in response to our prior conceptual engagement with numbers, and the needs that arise out of this engagement. The core of the number system out of which the numeral system grows is always the set of small numbers that we can conceptualize authentically. That is what anchors or grounds the system. That is the trunk, and the rest are the branches.

V.2 Logic and the Objectivity of Knowledge (1984)—The analysis Willard offers in a section of 1984's *Logic and the Objectivity of Knowledge*[65] is much the same as that in "Husserl on a Logic that Failed." However, he adds a nuance that is important for our discussion here. Specifically, he identifies "three distinct stages" in Husserl's investigation in *PA*, as Husserl attempts to answer the question, "*How* do the procedures actually employed in the science of arithmetic provide *knowledge* of the domain of numbers and number relations."[66]

The first stage is Husserl's "discussion of the intuitive acts in which small multiplicities or sets, and the corresponding small numbers, present themselves exactly as what they are, and are 'authentically' grasped."[67] However, "the normal arithmetical procedures cannot be illumined solely by an account of" such authentic ways of intending numbers. Thus, Husserl turns to the "*second* stage" of his investigation, which deals with inauthentic ways of intending numbers.[68] Yet, this stage also turns out to be inadequate, and Husserl moves into the third stage, realizing that

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[65] Willard, *LOK*, 110–18, but specifically 114–18.
[66] Ibid., 114.
[67] Ibid.
[68] Ibid., 115.

"there is required something essentially different from representations, whether authentic or symbolic." In fact, "[*m*]*ost* of what goes on in the practice of the arithmetician is not at all a matter of representing or conceiving of numbers," but is instead "a matter of working with one or more algorithms." Thus, Husserl's "account of arithmetical methods in terms of a *theory of representations* . . . failed him."[69]

I too have offered a three-stage analysis of *PA*, and yet have insisted on the continuity of the stages, and eventual success of the whole project. Husserl's theory of representations—or "presentations," or "concepts"—did not fail him, but instead led him to see the important role that signs play in our representations of numbers, and the way in which those signs come to form a system of their own. That system is justified by the underlying system of representations (concepts) out of which it grew, but can be used independently (as it were) of it. The theory of concepts or representations Husserl employed was not the last step in his journey toward understanding the contemporary practice of arithmetic, but it was an indispensable step and maintains its legitimacy even in light of Husserl's final conclusion.

V.3 Translator's Introduction to Early Writings *(1994)*—In 1994, Willard presented the first of his two major contributions to the translation of Husserl's early work. *Early Writings in the Philosophy of Logic and Mathematics* draws essays from various Husserliana volumes, and is designed to cover the years in Husserl's thinking and writing between *PA* and *Logical Investigations* (1900/1901).[70] In the translator's introduction, Willard takes up his discussion of *PA* once more.

[69] Ibid., 116.
[70] Willard, translator's introduction to *Early Writings*, vii–viii.

In the introduction, Willard writes that Part II of *PA* is "an attempt to respond to [the] question" of "*how* . . . a procedure with a formalized language or algorithm, in which consciousness of the relevant logical relations between concepts and propositions (or of their corresponding objects) simply plays no role, can supply knowledge of a domain left entirely out of consideration in the execution of the procedure."[71] Given that it is trying to solve this problem, Part II is "a rather strange piece of writing," Willard continues; in fact, it is "fumbling and inconclusive" because "at its outset its author still hoped to be able to explain how general arithmetic achieves its epistemic results by recourse to inauthentic representations *of numbers*."[72] The "lame conclusion" at which Husserl arrives in Part II is "that our unavoidable limitation to the use of symbolic number formations in the overwhelming majority of cases where we seek numerical properties, relationships and laws . . . is what forces us, for the purposes of knowledge, to develop the rule-governed symbol system that makes up general arithmetic."[73]

After completing Part II, Willard continues, Husserl then wrote his "Author's Report" [*Selbstanzeige*] about *PA*, which claims that Part II of *PA*, "already clears up the most elemental symbolic methods of number arithmetic, which are based upon the rigorous parallelism between the number concepts and numerals, and between the rules for compounding concepts to form judgments and the rules for compounding symbols to produce formulae."[74] However, Willard is skeptical. He responds, "What is *not* cleared up is the role of *calculation,* or formal transformation and derivation. Does it also work by some sort of parallelism, or what? Husserl does

[71] Ibid., xii.
[72] Ibid.
[73] Ibid.
[74] Husserl, *PA*, 287, quoted in Willard, translator's introduction to *Early Writings*, xii–xiii.

not know at this point."[75] Apparently as evidence for this claim, Willard notes that Husserl "closes the '*Selbstanzeige*' by saying that 'The higher symbolic methods, of a totally different type, which constitute the essence of the general arithmetic of number, are reserved to the second volume [of the *Philosophie der Arithmetik*], where that arithmetic will present itself as one member of a whole class of arithmetics, united through sharing identically the same algorithm'."[76]

Husserl's statements in the *Selbstanzeige* seem to me to be making a stronger claim than evidently they seem to Willard. "Calculation, or formal transformation and derivation," is—as we have already seen—part of "the most elemental symbolic methods of number arithmetic." Therefore, in answer to Willard's question, numerical calculation *is* "based upon the rigorous parallelism between the number concepts and numerals, and between the rules for compounding concepts to form judgments and the rules for compounding symbols to produce formulae."[77] Thus, what Husserl leaves for the never-published second volume of *PA* is not an explanation of basic numerical calculation, but instead a thoroughgoing exploration of what has already been discussed and adumbrated in the first volume: basic numerical calculation (involving the numerals for the positive integers, in addition to the signs for the four basic operations) is but one instance of calculation in general (which would involve signs for negative numbers, fractions, imaginary numbers, and so forth).[78]

[75] Ibid., xiii.
[76] Ibid., quoting Husserl, *PA*, 287.
[77] Ibid., quoted in Willard, translator's introduction to *Early Writings*, xii–xiii.
[78] "One can . . . conceive of calculation as any rule-governed mode of derivation of signs from signs within any algorithmic sign-system according to the 'laws'—or better: the conventions—for combination, separation, and transformation peculiar to that system" (Husserl, *PA*, 258 [ET 273]).

Willard then turns to what he says is "a much clearer explanation of [Husserl's] change in view" in his (Husserl's) letter to Stumpf.[79] Willard writes:

> This important letter was written at the point where the galley proofs for Part I of *Philosophie der Arithmetik* had been completed. [Husserl] hoped to complete the pages for Part II in the next nine weeks. But he confessed to his mentor and friend that he "possesses as of yet no coherent outline of the part dealing with *arithmetica universalis*," i.e., of Part II of the book, and can only hope that it will come to him 'in a rush', such as he had experienced with certain other writings.[80]

However, I would argue that we must note the following. First, Husserl tells Stumpf that he has the "guiding ideas"—indeed, he has a "theory"—in hand, even though he does not have the outline for Part II. Second, he says the "guiding ideas have been ripened and secured through years of painstaking deliberations" and "the theory" has been "developed over a period of years." Third, Husserl then proceeds to explain "[t]he results which [he has] obtained," saying they are "striking enough."[81]

> The opinion by which I [Husserl] was still guided in the elaboration of my *Habilitationsschrift*, to the effect that the concept of cardinal number forms the foundation of general arithmetic, soon proved to be false. . . . By no clever devices, by no "inauthentic representing," can one derive negative, rational, irrational, and the various sorts of complex numbers from the concept of the cardinal number.[82]

We have, of course, already seen this passage before. But we can now, at last, examine exactly what Willard believes it shows, and evaluate his conclusions regarding it.

Willard is of the opinion that the paragraph above shows Husserl's letter to Stumpf to have been written in 1891, after Husserl had already begun chapter 11 of *PA*. He bases this conclusion on the

[79] Willard, translator's introduction to *Early Writings*, xiii.
[80] Ibid., quoting Edmund Husserl, "Letter to Stumpf," in *Early Writings*, 13.
[81] Ibid.
[82] Ibid.

fact that Husserl explicitly appeals to Brentano's distinction between "*symbolic* and *authentic* representations" at the beginning of Chapter 11, where, in a footnote, he (Husserl) claims to "owe the deeper understanding of the vast significance of inauthentic representing for our whole psychical life" to Brentano.[83] Willard's reasoning is that if

(a) Husserl is still appealing to inauthentic representations in chapter 11 as essential for our understanding of arithmetical calculation,

(b) Husserl says in his letter to Stumpf that he has realized that Brentano's theory of representations will not be able to explain arithmetical calculation, since it cannot explain a wide range of number types which any full theory of calculation would have to include, and

(c) Husserl will claim in chapter 13 of *PA* that it is *mechanical* (not conceptual or representational) sign-manipulation that is the real heart of actual calculation, then

(d) Husserl's letter to Stumpf must have been written after Husserl had begun chapter 11 of *PA*, but before Husserl had written chapter 13.

The general idea we get from Willard's arguments over the years, then, is that Husserl writes *PA* up through the beginning of chapter 11, hits a conceptual roadblock regarding the inadequacy of Brentano's theory of representations, writes to Stumpf to describe this roadblock, and then finishes chapters 11, 12, and 13, including only an implicit acknowledgement of the difficulty he has

[83] Husserl, *PA*, 193 and n. 1 (ET 206). In an editorial footnote to the paragraph quoted above (p. 172), Willard writes: "On the immense significance of this comment for Husserl's development, consider the footnote on the first page of Chapter XI of the *Philosophy of Arithmetic*. It is unlikely that this footnote would have been published in 1891 if this letter were from the winter of 1890" (editorial footnote [n. 3] in Husserl, "Letter to Stumpf," 13).

encountered, and without including a complete solution to that difficulty. I would argue, however, that Husserl's letter to Stumpf tells a different story.

First, Husserl does not say in his letter to Stumpf that his realization about the inadequacy of Brentano's theory of representations was recent; rather, he says he came to that realization "soon" after 1887's *Habilitationsschrift.* If Husserl is writing to Stumpf in the "Winter of 1890 or 1891,"[84] and Husserl's *Habilitationsschrift* was published in 1887, then "soon" after the *Habilitationsschrift* would put the realization sometime in 1887 or 1888, around two years before Husserl's letter to Stumpf and long before he had begun to write chapters 11, 12, and 13 of *PA*.

Second, Husserl tells Stumpf the ideas he has for the rest of *PA* "have been ripened and secured through years of painstaking deliberations," and his theory has been "developed over a period of years."[85] He then sets out to elaborate on these ideas (and this theory) by going back to his original realization that he would need something more than Brentano's theory of representations. Instead of referring to this original realization as being a recent event, therefore, the context places "years of painstaking deliberations" and theory development *between* that realization and Husserl's letter to Stumpf. In support of this claim, here is the passage in context.

> About 200 pages of proofs[86] are now finished, and I still have 150–200 pages to complete in the next 9 weeks. If heaven does not now deny me the strength, and if the evil demon of nervousness also is subdued from here on, I hope then to bring it all to a happy conclusion. I confess that for this I possess as of yet no coherent outline of the part dealing with *arithmetica universalis.* However, the guiding ideas have been ripened and secured through years of painstaking deliberations. I unfortunately do not have the gift of first coming to clarity in the process of writing and

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[84] Willard, editorial footnote (n. 1) in Husserl, "Letter to Stumpf," 12.
[85] Husserl, "Letter to Stumpf," 13.
[86] Here Willard adds an editorial footnote, noting that Husserl is speaking of *PA* (Willard, editorial footnote [n. 2] in Husserl, "Letter to Stumpf," 12).

> rewriting. So long as I am unclear, the pen does not budge. But once I have come to a clear understanding, everything moves along rapidly. My *Habilitationsschrift* was likewise put into a finished text within a few weeks, without previous outline.
>
> So I also hope now — if I retain the happy disposition which has turned me into a new man this winter — to be able to lay out in one thrust, in the course of the next 9 weeks, the theory developed over a period of years. The results which I have obtained are striking enough. The opinion by which I was still guided in the elaboration of my *Habilitationsschrift,* to the effect that the concept of cardinal number forms the foundation of general arithmetic, soon proved to be false. (The analysis of the ordinal number already made this clear to me.) By no clever devices, by no "inauthentic representing," can one derive negative, rational, irrational, and the various sorts of complex numbers from the concept of the cardinal number.[87]

Husserl then proceeds to describe his findings and theory for the next three pages.

Thus, Husserl is not relating a recent discovery to Stumpf, as Willard argues. The realization that he would need something more than a theory of "representations" to explain and justify the practice of modern mathematicians did not come to Husserl after he had begun Part II of *PA*, but before he completed his work. Rather, we must conclude that Husserl set out to write Part II knowing basically where it would end up, even though he did not have a full "outline" in hand. As the writing of his *Habilitationsschrift* showed, Husserl does not need an outline; he just needs to be clear about the theory he is explaining.

Finally, it is clear from his letter to Stumpf that Husserl is already clear about the theory. As he concludes his discussion of the theoretical questions he had been working through over the past few years, Husserl writes this:

> The sign system of *arithmetica universalis* divides into a certain sequence of levels, comparable to that of a system of concentric circles. The lowest level (the innermost circle) is occupied by the signs 1, 2 = 1 + 1, 3 = 2 + 1, etc.; the next by fractional signs; and so on. The signs of the lowest level, and they only, are independent. Those of the higher levels are formally

[87] Ibid., 12–13.

> dependent upon those of the lower levels, and ultimately upon the lowest. To each domain there belong rules of calculation ("formal laws"). Those of the higher domains are dependent upon those of the lower, and include them formally. The rules of calculation are, then, so formed that each "equation" (in whatever way it may be set up, i.e., by means of whatever domain levels) is satisfied as an identity with reference to *the* signs and *the* domain of rules which it actually involves.[88]

Once we have defined the signs of the lowest level (the signs for the positive integers), we can then employ those signs in the definition of other signs (for example, fractions), and then employ those signs in the definition of still other types of signs. Further, the rules for calculating with the signs of higher levels are based on the rules for calculating with signs of the lower levels. Thus, all the signs for rational, negative, imaginary numbers, and so forth, come to be defined (as well as come to be utilizable in calculations) ultimately by reference to the signs for the positive integers (and the calculative rules governing those signs).

This does not mean, however, that Husserl believes that numbers simply *are* signs, versus Helmholtz.[89] Rather, the whole system of signs is grounded in concepts.

> Thus, if we are dealing with the domain of the cardinal *concepts,* and if I can prove that they and the elemental relations of the cardinal numbers admit of an adequate symbolization by means of the signs of that lowest range (that of the totality of whole numbers), then each correct sign equivalence ("equation") between the signs of this level represents a correct arithmetical proposition. But from this law it follows: For the discovery of the laws of number, in the genuine sense of the word, I can use the whole algorithm of the *arithmetica universalis*. (Notwithstanding that all of the remaining signs of the *arithmetica universalis* are 'senseless', admitting of no interpretation.) And so for all levels.[90]

What we must first do, then, is show how our numeral system "adequately symbolizes" the "cardinal number concepts" and "the elemental relations of the cardinal numbers." This is what Husserl

88 Ibid., 16.
89 Ibid., 14.
90 Ibid., 16–17.

does in Part II of *PA*. Second, we must provide the appropriate definitions of the signs for the "senseless" signs (for example, the signs for negative and imaginary numbers) in terms of that numeral system. This is what Husserl leaves to the never-published volume two of *PA*. The hard conceptual work—which employs Brentano's theory of representations—however, is done when Husserl finishes Part II. What is left for volume two is filling out the details of how the "senseless" signs not yet defined (as well as operations upon them) can be defined in terms of the meaningful signs (and operations upon them) of the numeral system. Volume two, therefore, would have been a mere fleshing out, or filling in, of the theory already on offer in volume one.

And, I would argue, this would not have been at all difficult to accomplish. Husserl has explained how mechanical calculation is grounded in conceptual calculation, and has already shown how calculation can proceed by employing memorized sign equivalences. Given what Husserl tells Stumpf about the "sign system of *arithmetica universalis* divid[ing] into a certain sequence of levels"—each of which is connected to the ones below and above it by "formal laws" involving sign equivalences (as stated through equations)[91]—filling out the rest of the theory in volume two of *PA* would have been something very close to busywork. Husserl would simply need to state explicitly the new sign-equivalences we would have to memorize in order to calculate with signs like "-1" and "$\sqrt{-1}$."[92] Volume two, therefore, is missing not because Husserl lacked the requisite theory to write it, but because its contents could be easily worked out from what is already implicit in volume one (and thus Husserl could turn to other, more pressing issues).

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[91] Ibid., 16.

[92] For example, "-1 = x – (x + 1)," "-1 + x = x – 1," "x – -1 = x + 1," "$\sqrt{-1}$ = [x – (x + 1)]$^{0.5}$," "($\sqrt{-1}$)^2 = -1," and so on.

Therefore, Husserl's realization that the theory of representations was inadequate did not lead him to abandoned the theory. Rather, it led him to incorporate something else into his account, *in addition to* the theory of representations. This is why Part I of *PA* uses Husserl's work from the *Habilitationsschrift*. The *Habilitationsschrift*'s basic idea—that we could understand arithmetical practice through Brentano's theory of authentic and inauthentic representations—was inadequate in the sense of being incomplete, not in the sense of being mistaken. It lays the necessary groundwork on which Part II builds.

V.4 Trnaslator's Introduction to PA(ET) (2003)—Willard is the translator of *PA* into English, and all Anglophone students of Husserl's early work owe him a debt of gratitude for this translation. In his helpful translator's introduction to the text, we find only a brief mention of the failure of Husserl's theory. However, what he does say shows that he still holds, after more than two decades, at least to the basic outlines of his original argument.

> [A] note of disappointment hangs over the conclusion of the book. . . . He has no general theory of arithmetical operations. The purely formal character of those operations is something that cannot be elucidated by a theory of "inauthentic" or symbolic representations. For while the symbols involved in formal calculation are, in a certain respect, symbolic representations, they are not utilized as such in the calculating. In calculating we do not think of what the symbols involved are symbols of. *How, in general, the formal operations of arithmetic work, Husserl cannot say*.[93]

I include this quotation because it so neatly encapsulates Willard's basic argument. Since arithmetic as we practice it is mechanical, we are not using the number signs as signs for numbers. Therefore, an understanding of number intentions (or "representations," or "concepts") will not help us understand what we are actually doing

[93] Willard, translator's introduction to *PA (ET)*, lvii.

when we practice arithmetic. This actual practice is something Husserl ultimately leaves unexplained.

Willard then continues.

> And there is a further embarrassment in the fact that within those operations or calculations there show up symbols that—if we did stop to think—could not receive any possible "real" interpretation in the domain of numbers, such as the square root of -1. These are the "imaginary" or "impossible" numbers that long haunted Husserl's efforts to understand basic arithmetic.[94]

By now, we already know how to respond to these claims. *PA* ends not in disappointment and embarrassment, but with a sense of accomplishment and the promise of further progress.

V.5 Conclusion—Willard's basic reading of *PA* articulates the text into two stages: one in which Husserl relies on Brentano's distinction between authentic and inauthentic representations to explicate arithmetical calculation, and one in which Husserl rejects that approach, and turns instead to a theory of signs. Willard first presented this analysis in his "Husserl on a Logic that Failed" and continued to propound it up through his translator's introduction to *PA(ET)*. However, in one passage from *Logic and the Objectivity of Knowledge*, Willard also presents the development of *PA* as progressing through three stages. The two analyses, however, are perfectly compatible. The three-stage analysis is simply the two-stage analysis, with the Brentano-dependent first stage being articulated into two parts.

Especially important for Willard's argument is Husserl's letter to Stumpf. Willard takes it to mark the turning point in Husserl's writing of Part II of *PA*, where Husserl is forced to move from the Brentano-dependent stage of his investigation to the Brentano-rejecting stage of the same. However, I have argued that

94 Ibid.

the letter does not show Husserl having to change course in the midst of writing *PA*. Rather, it shows Husserl discussing the conclusions toward which he was building part II of *PA* from the very beginning.

No one has done more than Willard over the past three decades to encourage scholars to grapple with *PA*, and thereby to give it a second look. However, I would argue that—inspired particularly by Willard's three-stage analysis of *PA*—it is now time that we give *PA* a third look. Furthermore, the passages we have examined above are merely parts of Willard's overarching project of defending the validity and continuing importance of Husserl's philosophy.[95] I hope, in other words, that in trying to save *PA* from Willard's analysis in the passages examined above, what I am actually doing is furthering Willard's own general mission.

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95 Willard's "Husserl on a Logic that Failed," is, in large part, an attempt to show that Husserl did not reject *PA* as being guilty of "psychologism" (see pp. 46 and 64 of that essay). In *LOK*, Willard argues that Husserl's "account of the problematic of the objectivity of knowledge, and the main lines of his solution, constitute a lasting contribution to the theory of knowledge. He seems to be in the main correct in his views of the nature of human knowledge" (x). In his translator's introduction to *PA(ET)*, Willard writes that Husserl is "one of the few truly great philosophical minds" (lxiii), and says that *PA* "contains . . . some of the finest work later characterized by its author as 'phenomenological description' to be found anywhere" (lxii). Also, see Willard's "Realism Sustained? Interpreting Husserl's Progression into Idealism," presented at the Franciscan University of Steubenville's conference, "The Early Phenomenology of Munich and Göttingen," April 29–30, 2011 (http://www.dwillard.org/articles/artview.asp?artID=151), in which he takes important steps in what he calls his "project of delivering Husserl from idealism." During the Q&A session following this presentation, Willard argued that the idea that Husserl became an idealist has unfortunately led people to take Husserl's philosophy less seriously. In contrast, he writes in "Realism Sustained?": "The possibility of recovering authentic knowledge of the amazing richness of manifold fields of being, including the human self and its knowledge, and especially the inexhaustible ideal realms of essence, resulted in a powerful surge of philosophical interest and activity among Husserl's younger associates. Indeed, the possibility of knowledge is tied very directly to the possibility of philosophy itself—which of course has been seriously in question among philosophers themselves for a century or so. If Husserl was right, there was hope." As he made clear in the Q&A session after the presentation, Willard believes that Husserl was right, and there is hope.

VI. Response to Hopkins

*VI.1 Introduction—*We now turn to chapter 13 of Hopkins's *Origin of the Logic of Symbolic Mathematics* (2011), which is entitled "Authentic and Symbolic Numbers in Husserl's *Philosophy of Arithmetic*."[96] In that chapter, published over three decades after Willard's original article, Hopkins summarizes his own argument as follows.

> The psychological and logical investigations in Husserl's *Philosophy of Arithmetic* contain foundational analyses of arithmetic that are guided by two distinct and incompatible theses. One is the thesis that the basic concepts and calculational operations of universal arithmetic, that is, the numbers and the algorithms of both ordinary arithmetic and the symbolic calculus, have their foundation in the concept of cardinal number. The other is the thesis that both the numbers and algorithms of universal arithmetic have their foundation in a system of signs that are not conceptual but rather formal-logical, in the sense of that part of logic defined as symbolic technique. The juxtaposition of analyses guided by conflicting theses in the single text of *Philosophy of Arithmetic* is explained by a radical shift in their author's view of the logical relation between authentic and inauthentic number "concepts."[97]

The description here of Husserl's "two distinct and incompatible theses" is familiar from what we have seen in Willard. This is a reflection of the fact that the conclusion of Hopkins's argument is, in all important respects, identical to Willard's. What is unique to Hopkins's analysis is found in the final sentence of the above quotation. Hopkins argues that it is Husserl's changing understanding of "the logical relation between authentic and inauthentic number 'concepts'" that leads him (Husserl) to change his thesis about arithmetic. It is the job of *Origin*'s thirteenth

[96] Hopkins, *Origin*, 105. This chapter was evidently available to some as early as 2003, since Willard makes reference to it in translator's introduction to *PA(ET)*, lxiv, n. 42.
[97] Ibid., 145.

chapter to provide evidence for this claim, and to explain exactly what it means.

Husserl begins *PA* with an understanding of authentic and symbolic presentations or concepts as being "logically equivalent." This means, at least in part, that symbolic presentations (or concepts) can have the same objects as authentic presentations (or concepts).[98] For example, we can authentically present or conceive of the number five, and also symbolically (inauthentically) present or conceive of the same number. The clearest statement of this belief in *PA* can be found at the beginning of chapter 11,[99] but Husserl presents it as a clarification of an assumption he had been making all along.

The problem with this assumption, Hopkins argues, is that Husserl introduces a kind of symbolic number presentation (or concept) that cannot be directed toward the same number as some authentic presentation.[100] This is because these symbolic number presentations are not directed toward the same objects as authentic presentations.[101] Furthermore, Husserl—according to Hopkins—eventually decides that all the relevant symbolic number presentations are of this latter variety, and thus all the relevant symbolic number presentations cannot be logically equivalent to any authentic number presentations.[102]

The type of symbolic number presentation (or concept) that Husserl introduces is what Hopkins calls the "signitively symbolic" numbers.[103] As opposed to the "numbers it is still possible to symbolize conceptually"—which Hopkins simply calls "symbolic

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[98] Ibid., 117–18 (n. 21), 123, 129, 135.
[99] Husserl, *PA*, 194 (ET 206–7).
[100] Hopkins, *Origin*, 117, 123.
[101] Ibid., 118 (n. 21), 123, 128, 138.
[102] Ibid., 128, 136, 139.
[103] Ibid., 118 (n. 21).

numbers" or "symbolic number formations" (terms he seems to treat as interchangeable)[104]—"signitively symbolic numbers" are "those which can *only* be symbolized *signitively*."[105] Signitively symbolic numbers are, according to Hopkins, mere numerals.[106] However, it seems that all symbolic numbers, or symbolic number formations, are mere numerals on his view,[107] especially since he claims the "elementary concepts" on which number systems are based—specifically, those out of which the base-ten number system constructs its systematic number concepts—"are the number signs

[104] "Husserl initially speaks about symbolic numbers 'as a matter of symbolic formations for those species of the number concept that are not accessible to us in the authentic sense' ([*PA*] 234). Accordingly, the signs involved in the symbolic number formations were conceived of as making 'possible, through a complex of indirectly characterizing (but themselves authentically presented) relative determinations, an unrestricted expansion of the domain of number' ([*PA*] 240)" (Hopkins, *Origin*, 116). "The correct way of considering the sensible signs involved in symbolic number formations and therefore of understanding the symbolic numbers is to recognize that sensible signs 'participate in a far more striking manner in our symbolic formations than we have asserted . . .' ([*PA*] 241)" (Hopkins, *Origin*, 116–17). "Husserl therefore makes the distinction, within the very domain of symbolic number formations themselves, between the 'numbers it is still possible to symbolize conceptually' ([*PA*] 242), numbers whose 'conceptual content could still be brought before the mind,' 'and those that can only be signitively symbolized'" (Hopkins, *Origin*, 117). To this Hopkins appends the following text in a footnote: "Husserl comes to distinguish between these two distinct 'concepts' of symbolic numbers" (ibid., 117, n. 21).

[105] Husserl, *PA*, 242 (ET 256).

[106] "It is clear, then, that for Husserl the 'external signs' that compose signitively symbolic number formations are not understood as 'general names'" (Hopkins, *Origins*, 129). "Strictly speaking (as we have seen), signitively symbolic numbers are not number concepts, according to Husserl, because their meaning as signs does not—as in the case of systematically symbolic numbers—accompany the concepts and conceptual operations of number formation 'whose conceptual content could still be brought before the mind' ([*PA*] 242)" (Hopkins, *Origins*, 142 [n. 36]).

[107] Hopkins refers to "the numerals composing symbolic number formations" (ibid., 116) and "the signs that compose the symbolic number formations proper to arithmetic" (ibid., 117), and says that "symbolic numbers are properly understood as general names that refer, albeit indirectly, to the segregation of contents and their collection" (ibid., 123).

(numerals) 1 through 9."[108] Hopkins, in other words, seems to hold that all symbolic numbers are, ultimately, signitively symbolic.[109]

Hopkins argues that "signitively symbolic numbers manifest a complete *conceptual* independence from the authentic *and* inauthentic number concepts."[110] In other words:

> Whereas . . . symbolic numbers ["that are 'symbolic' by virtue of their inauthentic presentation of multitudes"] are properly understood as general names that refer, albeit indirectly, to the segregation of contents and their collection, symbolic numbers that are symbolic owing to their function as the signitive "representatives" of precisely these symbolic numbers . . . do not refer to multitudes at all. . . . [T]his recognition amounts to the abandonment of the view expressed here that authentic and symbolic presentations are logically equivalent, which is the view that formed the point of departure for Husserl's investigation of the symbolic presentation of multitudes and numbers.[111]

Hopkins's claim here seems to be the following. If signitively symbolic numbers are signs for symbolic numbers, they do not refer to groups (nor, therefore, to numbers), and thus cannot be logically equivalent to either inauthentic or authentic presentations of numbers, since those presentations are, at heart, presentations of groups.[112] And since arithmetic turns out to be based upon signitively symbolic numbers,[113] arithmetic turns out to have no relation to number concepts, as Husserl had initially thought.[114]

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[108] Ibid., 130.

[109] "Husserl's analysis of symbolic numbers arrives at the conclusion that their logical content is inseparable from the external signs and the sign system by which they are designated. Consequently, the logical status of their symbolic content is now *signitive*, which means that they refer neither to the *concept* of a determinate amount of units nor to their symbolic idealization, that is, to the *conceptually* symbolic system of number formation that expands numbers beyond the psychological limits of their authentic presentation" (ibid., 128).

[110] Ibid., 146.

[111] Ibid., 123; my interpolation, using a quotation from the previous sentence in the text.

[112] Ibid., 123.

[113] Ibid., 114–15, 116.

[114] Ibid., 145.

Before going any further, however, we should examine the passage in *PA* from which Hopkins derives the notion of signitively symbolic numbers.

VI.2 Husserlian Interlude—In chapter 12 of *PA*, we find a section entitled, "Expansion of the Domain of Symbolic Numbers through Sense Perceptible Symbolization."[115] There, Husserl notes that we take ourselves to have access to an enormous range of numbers, beyond those that we can authentically present. We do not have to stop counting, for example, with twelve. We can continue on to twenty, one-hundred, five-hundred-thousand, ten-million, and so on, given the time and will. What is it precisely that gives us this ability?

At first, Husserl says, he claimed that it was our ability to symbolically present numbers according to the rules of some number system that allowed us to do this.[116] We cannot, for example, authentically present the number one-hundred, but we can employ the rules of the base-ten number system to split it into ten tens, and thereby symbolically present it. But what of much larger numbers, like one-hundred twenty-three thousand, four-hundred fifty-six? Even using the rules of the base-ten system, we would have to conceive of the number as *one* set of ten sets of ten sets of ten sets of ten tens, *two* sets of ten sets of ten sets of ten tens, *three* sets of ten sets of ten tens, *four* sets of ten tens, *five* tens, and *six* ones. And there is no way we can mentally hold together such a conceptual articulation.[117]

Even a number system, therefore, does not—on its own—give us conceptual access to all the numbers with which we seem to be able to work.[118] Instead, it introduces a numeral system to aid us,

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[115] Husserl, *PA*, 240–42 (ET 254–56).
[116] Ibid., 240 (ET 254).
[117] Ibid., 240 (ET 254–55).
[118] Ibid., 240–41 (ET 255).

and this numeral system provides the support framework on which we rely.[119] We employ the numeral system in two ways. When faced with a large, complex numeral (like "123,456"), "We obviously first think the mere concept: a certain number which corresponds to *this* sign complex." However, we can also investigate "the exact content of [this] concept" through "a chain of explications which has its support in the unity and special mode of the formation in the composition of the sign."[120] Or, to put it another way, we can go through the process of "interpretation," which involves a "chain of conceptual transformations."[121] These are what Husserl earlier in the section called "survey[ing] the sequence of linkages" between the parts of a large inauthentic number concept (for example, between the sets of ten within 123,456), or "advancing step by step" through the "relationships" between those parts,[122] which we cannot achieve for large inauthentic number concepts without the support of number signs.[123] Later, in the section, Husserl will call this interpretation of number signs, "bring[ing] to consciousness the full conceptual content of a complex number sign," and in the next chapter he will call it the "*conversion of . . . signs . . . into thoughts*."[124]

Thus, Husserl sees us as being able to deal with larger and larger numbers first by adding symbolic number presentations or concepts to our repertoire, and then by adding number signs to support our presentations or concepts of numbers. Nevertheless, "even now we are not absolutely unlimited as we follow the route of

[119] Ibid., 241–42 (ET 255–56).
[120] Ibid., 242 (ET 256).
[121] Ibid., 241 (ET 255).
[122] Ibid., 240 (ET 254).
[123] Ibid., 240–41 (ET 254–55).
[124] Ibid., 258 (ET 273). A parallel discussion, involving the signs for nonsystematic numbers, occurs on ibid., 224 (ET 237).

mere signs."[125] That is, as we construct the number signs for ever larger and larger numbers, we eventually reach a point where we can no longer conceptually and completely interpret the numeral we have just constructed (for example, "987,654,321,234,567,890"). That is, even with the help of the sign, we cannot conceive of the number in question in any kind of distinct way. The best we can do is, as Husserl said earlier, "think the mere concept: a certain number which corresponds to *this* sign complex."[126] And we can get no further. However, "we no longer feel the limits"; "we do not really notice at all the difference between the numbers it is still possible to symbolize conceptually, and those which can *only* be symbolized *signitively*."[127] This is because we rarely ever try to go more deeply into the meaning of the number signs with which we work than "thinking the mere concept: a certain number which corresponds to *this* sign complex," even when we could (because the numbers in question are "small").[128] We simply treat all numbers, no matter how large or small, as "whatever number corresponds to this sign complex."

VI.3 Evaluation—We now return to Hopkins. The argument in chapter 13 of Hopkins's *Origin* depends on there being a class of symbolic number presentations which are not directed toward groups of units, and which thus cannot be logically equivalent to authentic number presentations. Hopkins believes this criterion to be met by "signitively symbolic numbers," which he understands as (a) numerals that count as (b) symbolic presentations of numbers which, nevertheless, (c) do not actually present groups, and thus (d) cannot be logically equivalent to authentic number presentations. As we have just seen, however, Husserl's understanding of

[125] Ibid., 242 (ET 256).
[126] Ibid.
[127] Ibid.
[128] Ibid.

signitively symbolic numbers does not meet Hopkins's requirements. Signitively symbolic numbers are those numbers that are so large that we cannot conceptually interpret the numerals we employ in presenting them to any greater depth than, "whatever number this sign happens to stand for." Signitively symbolic numbers, therefore, are (a) not numerals. Signitively symbolic numbers, furthermore, are (b) not presentations of numbers; rather, they *are* numbers. Our presentations of signitively symbolic numbers, furthermore, do (c) actually present groups, insofar as they present numbers and numbers are the forms of groups. And finally, (d) our presentations of signitively symbolic numbers *are* logically equivalent to authentic presentations of those same numbers; it is just that our human minds are (contingently) not powerful enough to achieve the authentic presentations in question. The class of symbolic number presentations in *PA* to which Hopkins appeals, therefore, does not meet his requirements, and thus Hopkins's argument—namely, that Husserl is forced to change his understanding of calculation by his (Husserl's) introduction of a class of symbolic number presentations that intrinsically cannot be equivalent (in the logical sense) to some authentic presentation or other—does not succeed.

This does not mean, however, that Hopkins is wrong to hold that Husserl settles on a different understanding of calculation at the end of *PA* than we might have expected from the opening chapters of the book. But this, ultimately, is Willard's position repeated. And, I have tried to argue, the rhetoric of negation and disjunction employed by both (centering around terms like "abandonment," "failure," and "radical shift") does not match the progressive, organic, logically-motivated growth and development that actually characterizes the text of *PA*.

VII. Conclusion

In the foregoing we have seen two basic ways of reading Husserl's *Philosophy of Arithmetic*. The first, and the one I favor, takes the book to be Husserl's successful attempt to connect the contemporary practice of professional mathematicians—at least insofar as they practice arithmetical calculation—to our common experience of numbers. Husserl shows how three different types of arithmetical calculation can grow out of our common experience of numbers, with each type building on, and being stimulated by, the previous type. The second reading, favored by Willard and Hopkins, sees *PA* as involving two main attempts to describe arithmetical calculation as it is practiced by contemporary mathematicians. The first, which dominates almost the entire book, treats calculation as either an "authentic" or "symbolic" engagement with numbers, and ultimately fails. The second, which treats calculation as an engagement with signs alone, can be found toward the end of the book, and succeeds (at least to some extent).

What, however, is the ultimate import of all of this? First, if my analysis is correct, the ultimate import is *not* that the programs of Willard and Hopkins have been somehow refuted. As I noted above, I take myself in the present essay to be a coworker with Willard in his project of vindicating Husserl and promoting the Husserlian approach to philosophy. And, furthermore, I have only attacked the argument of one chapter in Hopkins's *Origin*. There are thirty-four other chapters in that extremely important work.

Second, if my analysis is correct, we are well on our way to recovering the proper understanding of the first major work of "one of the few truly great philosophical minds."[129]

[129] Willard, translator's introduction to *PA(ET)*, lxiii.

Third, I believe that Husserl's thought cannot be segmented into different eras (for example, the "prephenomenological," "phenomenological," and "idealist" eras), amongst which we must choose, looking for the one where he gets things right, while ignoring the others. Rather, Husserl's philosophy has an essential unity and coherence across the decades, and this essay has attempted to show that this continuity is not interrupted—or, indeed, eliminated—by a change of mind toward the end of Husserl's first book.

Fourth, and finally, if my analysis is correct, *PA*—as a whole—provides us with a coherent and compelling, if unfinished, philosophy of mathematics, on which phenomenologists and philosophers of mathematics can and should be drawing. It is a largely untapped resource, and we need not treat any of it as deuterocanonical.