

The Source of Platonism in the Philosophy of Mathematics Revisited:

Aristotle's Critique of Eidetic Numbers

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According to the so-called Platonistic conception of the nature of mathematical entities, mathematical statements are analogous to statements about real physical objects and their relations, with the one decisive difference that mathematical entities are neither physical nor individuated spatio-temporally and, thus, not perceived sensuously. Mathematical Platonism is therefore of a piece with Platonism in general, which posits the thesis of an ideal world of entities—*eidê*—that are both separate (*chôriston*) from and the cognitive and ontological foundations for the real world of physical things possessing spatio-temporal properties. While the nonidentity of the Platonistic conception of mathematical entities with the Platonism of the “historical” Plato is usually either tacitly or explicitly acknowledged by its proponents and critics alike, its connection with the “historical” Aristotle’s critique of Plato’s philosophy usually goes unacknowledged.

In what follows I want both to call attention to Aristotle’s connection with so-called Platonism traditionally conceived and to reconstruct a crucial aspect of his critique of the original Platonic *chôrismos* thesis that is missed unless the true target of this critique, the equally original Platonic account of eidetic numbers, is recognized. Aristotle’s critique of the Platonic separation of arithmetical beings from sensible beings has two foci, (1) the denial of the existence of pure arithmetical units independent of sensible

things and (2) the denial that there is any unity in a number of things. The first focus rejects the Platonic thesis of an “independent one” that is separate from the being-one of sensible things and therefore only accessible in thought. The second rejects the Platonic thesis that alongside the multitude of units that compose a number there is an ideal unity brought about by an ideal number (*arithmos eidetikos*).

Attentiveness to the two foci of Aristotle’s original critique of Platonism reveals that the traditional Platonistic conception of mathematical entities is informed exclusively by an abbreviated formulation of Aristotle’s critique’s first target, namely its thesis that the true subject of mathematical statements are mathematical entities that exist independently of physical entities. Situated within the context of the “historical” Aristotle’s critique, the so-called Platonistic concept of “mathematical entity” elides both the multitude of pure units that for Plato as for Aristotle is the subject matter of arithmetic and the dispute over the multitude’s proper manner of being that sets the terms for Aristotle’s critique of Plato and the Platonists. The second focus of Aristotle’s critique, which simultaneously presents an unrelenting polemic against the Platonic thesis of eidetic numbers together with the most detailed report available about this thesis itself, therefore contains the key to the original content of mathematical—or, more precisely—arithmetical Platonism.

In the interest of attempting to address, if nothing else, the original phenomenon of this Platonism, and, therefore, the source of Platonism in the philosophy of mathematics, the following remarks on Plato’s account of eidetic numbers and Aristotle’s critique thereof are offered.

Plato's Socrates' Dialectic

Plato's Socrates steadfastly refuses to relate to his interlocutors "what manner of power dialectic has, and exactly what *eidê* it's divided into, and what paths they take" (*Republic*, 532de), because to do so requires going beyond any image of the intelligible, and thus beyond the *logos*, to the truth itself as it is apparent to him. Socrates does not quibble over whether this truth appears to him in its very being or not, since this is undeserving of confident assertion, but he does maintain "that there is some such thing to see is something one can be sure of" (*Republic*, 533a). Rather than endeavor to leave the realm of images entirely behind, which, it should be stressed, there is no lack of willingness on Socrates part to do, the first ("Socratic") account of the *eidê* in the dialogues limits itself to the "prelude to the song" (of the "mode of passage" most proper to dialectic). The Socratic prelude focuses on the "know-how" (*technê*) requisite for the "conversion" (*metastrophê*) and "redirection" (*periogôgê*) (*Republic*, 518d) of the whole soul from the things that are becoming to what truly is. Plato's Socrates identifies this know-how with the lowly *technê* of counting and calculation used by all who count things and reckon with the resulting sums to solve problems (whether practical or theoretical) of multiplication and division and to study numerical ratios. Socrates stresses, however, "no one uses it rightly, as something suited in every way to draw someone toward being" (*Republic*, 523a).

Counting and calculation, used rightly, "draw" and "redirect" the soul toward being and the truth itself by forcing it to exercise its highest power of thinking, *nous*, first to clarify opposite sensations that strike the senses and then to contemplate and study the nature of the "pure" numbers employed by those who are formidable in arithmetic (mathematicians).

Opposing sensations (for instance, large and small) reported by the same sense (sight in this case) in the same thing (for instance, in the perception of each one of three fingers) awaken in the soul the “activity of *nous*” (*noêsis*) and counting, in order “to examine whether each of the things passed on to it [large and small] is one or two;” and, if “they appear to be two,” to grasp that each “appear as something distinct and one.” In the case at hand, *nous* will grasp that “each is one and both together are two,” and thus “be grasping the two as separate, because it wouldn’t grasp inseparable things as two but as one” (*Republic*, 524bc). Sight sees large and small mixed together while *nous* achieves clarity about this by grasping each as one and both as two, and what sight sees is called “visible” and what *nous* grasps is called “intelligible.”

Number and its source (*archê*), the one,¹ also appear to sight with something opposite to them, and therefore they, too, lead to the contemplation of what is and redirect the soul to that. Not only does sight see the same thing at the same time as one and unlimitedly many, but even more so does this happen with number. Indeed, contemplation of the nature of numbers is touted by Socrates for its “ease of redirecting the soul itself from becoming to truth and being” (*Republic*, 525c), especially when the numbers studied are not those “that have visible or tangible bodies” (*Republic*, 525d), but the sort made of parts in which “each and every one is equal to every one without even a tiny difference, and with none having any part within itself” (*Republic*, 526a). The study of these numbers “obviously forces the soul to use *nous* itself directed at the truth itself”

¹ Number (*arithmos*) in the ancient Greek context is a multitude of perceptible or intelligible “ones.” Two is therefore the first number and “one,” which (when originally combined in counting) composes the parts of each number, is not a number (because it is not a multitude) but the *archê* (source) of number. Because each single number is composed of an exact amount of “ones,” it appears to the soul as both one and many and thus, as a mixture of opposites.

(*Republic*, 526ab) because they are “things that only admit of being thought.”

Plato’s first, “Socratic” account of the *eidê* in the dialogues stops here, at the “prelude” to the song of dialectic and its mode of passage to and beyond the *eidê*. The Socratic endeavor to use dialectic’s “power” to redirect soul toward being and the truth itself therefore remains shrouded in a darkness that is ultimately mythical. Regarding the answer to the question “why?” the sharing in an *eidos* of things is the cause (*aitia*) for the being of each one of them, Socrates is not ready, “as yet” (*Phaedo*, 100d) to state with confidence whether it is on account of an *eidos*’ “presence” (*parousia*) in them or on account of its bringing about a “community” (*koinônia*) among them. His identification of sharing or participating (*methexis*) with “imitation” (*mimêsis*) does not clarify this matter either because Socrates’ account of the image-original relationship makes it clear that the *eidê* that function as the originals cannot (like sensible originals) be perceived independently of the images in the *logos* that reflect them. Thus, at the very least, the Socratic account of the image-original relationship in “imitation” is paradoxical, because both the image’s likeness to its original and the original’s greater degree of “beingness” (*ousia*) cannot be established by perception and, therefore, by the “investigation of nature.” And, at the very most, the account is (as Aristotle will argue) “without content,” as it speaks in “poetic metaphors” (*Metaphysics* A, 991a22-23).

Plato’s Socrates therefore does not follow the “push” of his own *logos* to pursue the dialectical mode of passage to its transimaginal end in accordance with his own stipulations regarding the “right use” of the contemplation of numbers’ nature to lead the soul to being itself and truth. Nowhere is this more apparent than in the

myth of recollection's account of the origin of learning and, therefore, of the origin of the soul's *logos* about the *eidê*.

Plato's Arithmological Account of the Eidê

The dialogues' second account of the *eidê* is neither readily apparent nor Socratic. In place of the dramatic figures of the philosopher Socrates and various nonphilosophers, the figure of the unnamed philosopher from Elea (the "Stranger") and the accomplished mathematician Theaetetus pursue to its end the "right use" of the *technê* proper to counting and calculation for redirecting the whole soul to the source of being and truth. In other words, they complete in deed what is merely prescriptive in the Socratic account of the *eidê*. Their dialectically imageless investigation of the *eidê*, and, more precisely, of the "greatest" (most original) "kinds" (*genê*²), is presented in a manner in which the inexactness of the images belonging to Socratic myth is superseded by the exactness of number and the one. Each of the five *genê*, "itself by itself" (*auto kath' auto*), that the "intelligibility" of any *eidos* presupposes, is examined in an investigation guided by the "nature" of the pure numbers employed by those who are proficient in counting and calculation. However, even with this added exactness, the true manner of being of the *eidê* is proven to exceed the *logos*' ambit, for the simple but profound reason that the intelligible units presupposed by the "know-how" of

² "Genê" is the plural of "*genos*," which is derived from "*gignesthai*" and means coming to be and being born. "Genos" means both a group whose members share a kinship through birth (family) or generation (tribe) and the "common look (*eidos*)" characteristic of their members. "*Genos*" is therefore often employed by Plato in a way that is interchangeable with "*eidos*," although in his dialogue titled *Sophist* the "greatest *genê*" are more often than not referred to as "*genê*" than "*eidê*." While this dialogue lays the philosophical foundation for the technical distinction between (in their Latin translations) "genus" and "species," this distinction is not at work in the dialogue.

arithmetical counting to compose the numbers it counts with are naturally unsuited to count the intelligible units that dialectic must presuppose in its imageless investigation of the most original *eidê*.

Aristotle's report about Plato's "unwritten teachings" (*agraphois dogmasin*), which includes a polemic against Plato's apparent teaching that "the *eidê* are numbers" (*Metaphysics* M, 1086a12-13; *Metaphysics* N, 1090b32-33), provides an indispensable context for finding in some of the dialogues themselves veiled references to a genuinely Platonic teaching about "eidetic numbers." This is especially the case in the *Sophist*, where a philosopher and mathematician investigate dialectically the "five" most original *noêta* presupposed by thought whenever speech understands something, namely, the *genê* Motion, Rest, Being, the Other, and the Same.

The Community (Koinônia) of Eidê

The *Sophist's* discussion of the five greatest kinds is informed by the strange status of the "common thing" (*koinon*) exhibited by the mathematical numbers that are among intelligible objects in comparison with the "common thing" exhibited by nonmathematical intelligible objects. The manner in which the *eidê* of justice and health are shared by more than one thing and are thus common to each of the things that share in them contrasts sharply with the way the things that share in the *koinon* of a mathematical number are related to what (number) they have in common. Whereas justice and health characterize the many things that share in the *eidōs* of each, the many things that have a mathematical number in common are not characterized by the number they have in common. Thus while Socrates and Hippias both share in the *eidōs* health and are each healthy, taken together they also have being two in common,

although each of them is not two but only one. When it comes to intelligible objects like the *eidōs* health, what each of them is both are as well, whereas when it comes to intelligible objects like number, what each of them is they both are not (*Hippias Major*, 300a-302b).

The strange *koinon* character of number therefore exhibits a structure that presents an obstacle for any thinking that presupposes that all things that share in intelligible objects must also have these objects in common. The obstacle is the “impasse” (*aporia*) that follows the recognition by thought that numbers are intelligible objects for which this presupposition does not hold. Each of the items united by a number is different from the common thing that composes the number and conversely, the number is different from the items that it unites. This *aporia* provides the mathematical key to the Stranger and Theaetetus’ investigation and account of the five greatest *genē*, because their investigation shows that the presupposition that things must have in common what it is they share in does not hold for these most original intelligible objects either.

Beginning with the first three of these objects, Motion, Rest, and Being, they investigate what philosophers who presuppose that the whole of Being is composed of two elements (Motion and Rest) address when they say that both and also each of them *is* (*Sophist*, 250a-d). In saying that Motion and Rest *are*, they cannot be saying that Motion is identical with Being, because this would presuppose that Rest is not something that is. Likewise, they cannot be saying that Rest is identical with Being, because then Motion would be presupposed not to be. Is Being then addressed as a third thing beside or outside of both of them? This is impossible, as it would presuppose that neither Rest nor Motion *is* and that Being, according to its nature, is neither one (Rest or Motion) (*Sophist*,

250d). Something at rest or in motion is understandable as something that *is*, whereas something neither moving nor resting is not understandable at all as something that *is*. Thus, if it is presupposed that something that shares something in common with another thing must also be characterized as that thing they have in common, it would follow that Motion (itself by itself), by having Being (itself by itself) in common with Rest (itself by itself), would be *identified* as Being (itself by itself), and, conversely, that Rest (itself by itself), by having Being (itself by itself) in common with Motion (itself by itself), would be *identified* as Being (itself by itself). But this is precisely what cannot be presupposed when it comes to understanding what is, that either one of two things that are most opposite to one another, *is* what the other one is. If this were presupposed, then what is moving would be resting when it *is* and conversely, what is resting would be moving when it *is*.

Rather than attempt to resolve this *aporia* of Being, however, the Stranger proposes to Theaetetus that they let the matter rest there and investigate the ways of being together—“community” (*koinônia*)—of the *genê* Motion, Rest, and Being (*Sophist*, 251d-253c). The Stranger articulates three: no relation whatsoever between them, all being mutually related, or some being related and some not. The results of the discussion so far rule out both the complete lack of relation between these *genê* as well as all of them being mutually related. They are related because both Motion and Rest *are*; they are not all mutually related because Motion and Rest, being opposites, are not related in the precise sense that something at rest cannot remain what it is and be in motion and, conversely, something in motion cannot remain what it is and be at rest. Thus, the relations between the *genê* Motion, Rest, and Being are partial: Motion and Rest, as opposites, do not have a *genos* in common and are thus unrelated; however, because each *is*, they are nevertheless

in relation to the *genos* Being. Being, as the *koinon* that unites Motion and Rest, does so in a manner that permits their conjunction as opposites in kind without the terms of this conjunction being based in something that is common to each such that either would be an identical part of what they have in common. Motion and Rest therefore both *are* without Motion having something in common with Rest and conversely.

The partial *koinônia* between Motion, Rest, and Being exhibited by the Stranger and Theaetetus' investigation brings to light a decisive difference between Being's *koinon* character and that of number, despite the similarity that neither the units that are in community in a number nor the *genê* that are in community in Being can be characterized in terms of the *koinon* that brings them together. The intelligible objects brought together by the number two are not two, since each is exactly one; and the intelligible objects (*genê*) brought together by the *genos* Being are not Being, but precisely Motion and Rest. In this regard they are similar. But while the units that share in the *koinon* of a number do not differ from one another and are therefore equal, the *genê* that share in the *koinon* of the *genos* Being are not only different but they are most different, opposite.

Eidetic Numbers

Aristotle's account of what Plato said in his "unwritten teachings" about eidetic numbers addresses precisely the difference noted here between the *koinônia* that number brings about and that which the *genos* Being effects. Aristotle reports that Plato distinguished eidetic from mathematical numbers on the basis of their units. Each mathematical number is the unity of a multitude of unchanging units that are alike, while each eidetic number is the unity of

unchanging units that are unique (*Metaphysics A*, 987b19). The units of mathematical numbers are therefore comparable and capable of combining indiscriminately into any community with one another, while those of an eidetic number are incomparable (*asumblêtoi*) (*Metaphysics M* 7) and therefore incapable of combining into just any community with each other. Thus the *koinon* of the *genos* Being (as a whole) that unites a plurality of *genê* (Motion and Rest) exhibits an analogy with the *koinon* of mathematical number (as a whole) that unites a plurality of *units* (*monads*). In both cases, the unity of a multitude is provided by the *koinon* of the whole without that whole being partitioned by the multitude: for example, the number two unites units without being partitioned in these units, as each unit is exactly one, not two; so, too, the *genos* Being unites *genê* without being partitioned in these *genê*, as each *genos* (Motion and Rest) is (by itself) precisely not *Being*. However, the analogy between the *koinônia* of the units united in a mathematical number and that of the elemental *genê* united by the *genos* Being breaks down on the following point. The whole that composes the unity of a mathematical number is not attributable to the units in the multitude that it unites; “two,” for instance, is an attribute of neither of the units that are united by the number two, because each is precisely one, not two. The whole that composes the unity of the *genos* Being, however, is attributable to the *genê* in the multitude that is united by this whole, because Being is only understandable if Motion and Rest are presupposed to be—without, of course, either being identified as Being itself.

If it is presupposed that the *genê* Motion, Rest, and Being, as intelligible objects, are “units” with the same manner of being as the units in mathematical numbers, their number (*arithmos*) would be “three.” Motion would be one, Rest another one, and, finally, Being a third and final one. But precisely this presupposition was ruled

out when the presupposition that the *genos* Being is a third *genos*, and therefore completely separate from the *genê* Motion and Rest, was rejected, because it would follow from this that neither Motion nor Rest would *be* (and therefore share in Being). When it comes to Being, the Stranger and Theaetetus' dialectical inquiry makes its *koinon* manifest as precisely both Motion and Rest together. The multitude of *genê* that compose Being, therefore, cannot be accounted for on the basis of the presupposition that these *genê* are intelligible objects like the multitude of ones that compose a number. On this presupposition there are "three" kinds, Motion, Rest, and Being, whereas the (Stranger and Theaetetus') dialectical examination of this presupposition leads to the better presupposition that Being is not a third kind, but precisely both Motion and Rest, together. The presupposition that Being can be made intelligible by employing mathematical numbers therefore cannot withstand dialectical scrutiny, even though Being has a partially "arithmetical" structure, because, like the whole of number, the whole of Being retains its integrity and is not divided among its parts. The nonmathematical but nevertheless "arithmetical" unity that Being is shown to possess in the *Sophist* exhibits the structure of an "*arithmos*" of intelligible objects whose units are *eidê*, which confirms Aristotle's report that Plato distinguished mathematical numbers and *eidê* composed as numbers (*Metaphysics* A, 987b15-19; *Metaphysics* M, 1086a12-13).

*The Original Participation Problem
and the So-Called
"Separation" (Chôrismos) Thesis*

Plato's second account of the *eidê* provides an answer to the question that remains unanswered in his "Socratic" account, about

whether the single things in the multitude that “participates” in an *eidōs* do so because of the *eidōs*’ “presence” in them or because the *eidōs* brings about a “community” among them. Participation in the “itself by itself” status of a *genos* is the result of the single things that participate in it being brought together in a manner that establishes their “community” with the “common thing” that characterizes the *genos*. Because this “common thing” is not partitioned in the single things that it establishes as belonging together, the presupposition that it brings them together by being “in” them has to be rejected. The better presupposition is that a *genos* (as a whole) composes the parts that belong together with it (as *its* parts) in a manner that is analogous to the way a number (as a whole) composes its parts together.

Plato’s second account of the *eidê* is best characterized as “arithmological” rather than “arithmetical,” in recognition of the nonmathematical nature of the units that are united as an “*arithmos*” in an eidetic number (*arithmos eidetikos*). Plato’s arithmological account of the *eidê* therefore replaces the Socratic account’s metaphorical *logos* of participation with the Stranger and Theaetetus’ account’s arithmetical *logos*. In place of the vague or empty talk of the single things in a multitude “sharing” or “imitating” the *eidōs* that is responsible for their name and being, the talk about participation acquires the kind of precision that only the exactness of mathematical number and the (mathematical) one can bring about. This precision, however, is bought at a steep price. On the one hand, it concerns the participation of intelligible objects with one another, both the *eidê* presupposed in the dialectical passage and the most original *genê* presupposed by the being of every *eidōs*. Thus it leaves unaddressed the participation of the noneidetic things in the *eidê* that are responsible for their being. On the other hand, it focuses on the inability of *logos* to account for the

manner of being of the *eidê* by counting them. Not being countable places the manner of being proper to intelligible objects beyond the power of *logos* to provide a complete and a completely clear *account* of the “beinghood” (*ousia*) of the things in the intelligible realm. Moreover, because these things (*eidê*) are responsible for the being of the things in the visible realm, *logos*’ limit in this regard extends beyond the things in the intelligible realm to those visible things made intelligible by *eidê*.

The greater precision of Plato’s second account of the *eidê* over his first thus does not afford it any ultimate advantage in articulating the truth of the “end” (*telos*) of the intelligible realm that (as beyond Being) is the “source” of everything that is. But by “resolving” in the intelligible realm the “participation” problem first encountered in the visible realm, it does permit the relation between the *eidê* of the visible and intelligible realms themselves to be discerned with greater clarity. Just as the *genos* Being (itself by itself) is “separate” (*chôriston*) from the *genê* (Motion and Rest), so, too, is the intelligible realm (as a whole) “separate” from the multitude of things in the visible realm. Therefore, what Plato’s so-called “separation” (*chôrismos*) thesis addresses most originally is the “arithmos-structure” proper to the *genos*’ manner of being, which is responsible for composing elements that are fundamentally opposed into parts of a more comprehensive whole. Because participation in this sense is undeniably closer to the “source” of what is than participation in the sense of a multitude of things sharing in the unity of an intelligible thing, the latter sense of participation (of becoming with its opposite, Being) may be said to manifest an image of the original participation. In this, and only this sense, is it legitimate in the end to speak (Platonically) of participation as a form of “imitation.”

The common understanding of participation, which identifies it with things in the realm of becoming “imitating” things in the realm of being, is therefore very wide of the mark of how “Platonic” participation looks when the Socratic account of it is dialectically “united” with the arithmological one. Rather than a “two world theory” of separate realms of becoming and Being that are related as an image to its original, precisely the whole of the image-original relationship between becoming and Being is itself seen as an image and therefore imitation of the whole of the original *methexis* proper to the “community of *eidê* (or *genê*)” (*koinônia ton eidon*). As an image of the original participation relationship, the participation of the things that *become* in the things that *are* is a distortion of this original relationship. Both the name and the being of the thing that becomes has its source in an *eidos*, which lends itself to the belief that the *noêta* (*eidê*, *genê*) are countable and otherwise related in the same way as the names of the things in the realm of becoming that “share” in them are related to those things. But precisely this presupposition must give way, at the pinnacle of dialectical investigation, to the *nous*’ seeing of the uncountable but nevertheless “arithmological” manner of being proper to the “greatest kinds.” At this, the final, level of insight, the *noêta* presupposed by the *logos* (*eidê* and *genê*) are not numerically related as the silent or audible words (*onomata*) that correspond to them are. *Logos*’ counting of the latter (*onomata*) in order to comprehend the former (*eidê* and *genê*) is therefore misleading.

*Aristotle’s Dispute with Plato over the Manner of
Being Proper to the Eidê*

Aristotle’s dispute with the Platonic account of the *eidê* takes issue with its “separation” of the “beinghood” (*ousia*) belonging to a *genos*

from the multitude of single things that are encompassed by it. The answer to the question “why do things look the same” for Aristotle is not because there is an *eidos* of a highest rank, the *genos*, that encompasses everything that is, but because each one of the things that are is generated in some material (*hulê*) by an *eidos* whose proper mode of being is “being-at-work (*energeia*)”³ on it. The “beinghood” of things caused by the perpetual “being-at-work” of the *eidos* (without it ever undergoing change) is perceived and understood by the soul because the very “being-at-work” of an *eidos* responsible for generation also shapes the soul’s perception and informs its understanding. The “beinghood” or, more properly within the context of Aristotle’s metaphysics, “thinghood” of things has its source in “nature (*phusis*).” Aristotle’s account of the *eidê*, then, remains tied to the “first journey” (the investigation of nature) given up by Plato’s Socrates. This is even the case when the *eidê* involved are mathematical objects, because their “abstracted” manner of being is no more capable of existing in separation from the multitude of sensible (natural) things than the *eidê* of chickens or humans or any other natural being.

Aristotle’s account of the *eidê* thus takes issue with the two key tenets of Plato’s Socratic account of them and the major tenet of the arithmological account, all of which presuppose that an *eidos*’ proper manner of being is accessible through a multitude. In the case of the Socratic account, the claim that the *eidê* are “patterns (*paradeigmata*)” of the things that participate in them is rejected by Aristotle because (i) it does not address “what is the thing that is at work (*ergazomenon*), looking off toward the *eidê*” and (ii) it

³ “*Energeia*” is derived from the Greek neuter adjective “*energon*,” meaning “at work right now.” Its traditional Latinized translation as “actuality,” as the state or condition of acting or doing, obscures both the immediacy as well as “concreteness” of an *eidos*’ manner of “being-at-work,” which is always working on some “material” (*hulê*), the “product” of which is always something single.

presupposes that something can only become like something else by “being an image of it” (*Metaphysics* A, 991a21-25). In the case of the arithmological account, Aristotle rejects the view that in the relationship between the two causes of things (the independent One and the indeterminate dyad) the “*eidos* generates only once” while making “many things out of this material [the dyad]” (*Metaphysics* A, 988a3-4). Generation “surely” happens in the “opposite way” (*Metaphysics* A, 988a1), with the *eidos* being the cause of the generation of many things, and the material generating only once. And, in both cases, the view that an *eidos* is capable of *existing* in separation from the multitude of things is rejected, albeit for different reasons. In the Socratic account, it is because the cause of each thing’s motion remains unaddressed when its *ousia* is posited as *existing* apart from it. In the arithmological account, it is because the source of the one is a measure that originates in sensible things, which rules out Plato’s account of the source of a mathematical number (and, by extension, an eidetic number) being one because of the separate existence of a number’s generic unity.

*Aristotle’s Critique of
Plato’s Socratic Account of the Eidê*

Aristotle’s critique of Plato’s Socratic account of the *eidê* does not reject outright the claim that the *eidê* are “shared in (*metexein*)” but only that the things that “signify (*semainei*)” “thinghood” are different in the case of *eidê* and the many things that share in an *eidos* (*Metaphysics* A, 990b32-991a1). There is nothing “apart (*para*)” from these things, no “one-over-many” (*hen epi pollôn*), but rather there is something “common (*koinon*)” between them and their *eidos* (*Metaphysics* A, 991a2-4). The *eidê* are the *ousia* of things in the sense that the pairs that partake of the “Two itself”

(*Metaphysics A*, 990b33) are the same thing (that is, dual), no matter whether they are destructible or everlasting. Being “dual” is common to both the intelligible object and the things that share in it, which means that the “*eidos* [of the Two itself]” is the “same” as the pair, the two things, to which it is applied. Sharing in an *eidos* therefore does not duplicate the world, because the things that share in it do so by virtue of that which they have being, and this being is the “same (*tautôn*)” in the thing as in the *eidos*. The “Two itself” is no more dual than the pair of things that share in them, because it is no more dual “as applied both to itself and to something” (*Metaphysics A*, 991a4-7).

*Aristotle's Critique of Plato's
Thesis of the Separate (Chorismos)
Being of the Unity Belonging to the Genos*

Aristotle's account of the origin of mathematical numbers presents a fundamental critique of Plato's arithmological account of the unity of number having a source in a *genos* that is separate from the multitude of units composing each number. Aristotle's account is therefore only comprehensible within the context of Plato's arithmological account of the source proper to the unity of mathematical and eidetic numbers, as it explicitly takes issue with the *chorismos* thesis, the attribution of a *generic* unity to either kind of numbers and the necessity of the supposition of a nonmathematical one (*hen*) in order to ground arithmetically mathematical unity. Aristotle's “dispute” those who speak of mathematical things “as separate from sensible things,” however, is *not* whether they have being, but, rather, “the manner of their being” (*Metaphysics M*, 1076a34-38). Number, characterized as the discrete delimitation of a field of units into definite amounts, the

source (*archê*) of which is one (see *Topics Z* 4, 141b5-7; *Metaphysics A*, 1056b23), is therefore not in contention in Aristotle's critique of Plato's account of the unity of number.

For Aristotle, then, it is Plato's determination of the manner of being of "pure" numbers as independent of sensible beings that is in dispute, because it misses precisely the dependence on sensible things that is characteristic of each number. From the fact that it is possible to articulate the parts of something in declarative speech (*toi logôî*) before denominating the whole, it does not follow that the "thinghood (*ousia*)" of these parts has priority over the being of the whole (*Metaphysics M*, 1077b1-2). Likewise, it does not follow from saying that there is a number of something that this number exists outside of that which it delimits with respect to its definite amount. For example, in calling a human being "white," no other being is meant than precisely this white human being (*Metaphysics Z*, 1029b13-14). Likewise, in saying "three trees," "three" has the same status as the "white"; the definite amount of trees, namely, "three," therefore has no proper "nature (*phusis*)" (*Metaphysics M*, 1080a15; 1082a16; 1083b22).

For Aristotle, then, the status of the being of numbers is determined by their natural meaning: the assertion that certain things are present in a specific number means only that *such a thing* is present in just this definite multitude (*Physics D* 12, 221b14-15). This characterization of the manner of being number, however, presents the problem of how to account for the purely intelligible quality of mathematical numbers. This is a problem for Aristotle because, unlike Plato, who posits the independence of the intelligible being of the unit from sensible beings, Aristotle's reliance on the natural meaning (revealed in the analysis of ordinary speech) of numbers precludes the supposition behind Plato's position. It precludes the *hypothesis* that the homogeneous, indivisible (and

therefore unchanging) characteristics proper to the unit as the *archê* of mathematical numbers have their basis in a manner of being that is separate from sensible things. Aristotle, instead, articulates the manner of being belonging to these characteristics as arising “from abstraction (*ex aphairéseôs*)” (*Metaphysics* K, 1061a29), from being “lifted off,” “drawn off,” or, in other words, from being “abstracted” from sensible beings. The mathematical objects (*ta mathêmata*) studied by mathematical *epistêmê*, which in their being are not detached from sensible beings, are therefore nevertheless studied *as if* they were detached or separated. In line with this, Aristotle holds that “each thing may be viewed best in this way—if one posits that which is *not* separate *as separate*, just as the arithmetician and geometer do” (*Metaphysics* M, 1078a21-23).

How is it that someone who thinks mathematical objects is able to do so as separate from sensible beings, even though they are not separate? Aristotle’s answer to this question arises by considering how the “single parts (*merê*)” of sensible beings are gotten hold of in *logos*. When the aspects of a sensible thing are distinguished in speech one after the other from the concrete context of their being, a context without which they would not exist, for example, “this” “round” “white” “column,” it is apparent that the nexus of being that links all the parts together is disregarded in a manner that allows each part to be singled out and apprehended separately. This “disregarding of” establishes a new mode of seeing that allows something *in* sensible beings to come before its regard in a manner that, for all their variety and transitoriness, is unchanging. As such, it remains always in the same condition and therefore satisfies the demand that for Aristotle as for Plato must be satisfied for a being to be an object of *epistêmê*.

The lifting off characteristic of abstraction expresses nothing other than the “disregarding of” that makes possible the articulation

in *logos* of the single parts of a sensible thing, a disregarding in which sensible beings are deprived of their sensible qualities and individual differences. In a manner of speaking they shrink, becoming mere independent parts of bodies or mere bodies themselves, such that a demonstrative discipline becomes possible, one that, as it were, “reads off” such independent parts or bodies their arithmetical and geometrical aspects, namely, how many or how extensive they are. When the theoretical mathematician, moreover, in making that which comes into view in abstraction as the subject matter of study, no longer views what has been abstractedly lifted off as having its basis in mere bodies but views it as a part of them that in its own right is independent, such parts become “neutral” monads. Thus, the mathematician “studies things after having stripped away everything perceptible . . . , and this leaves behind [in the case of arithmetic] only what is of some amount” (*Metaphysics* K, 1061a28, 32-33). To posit that mathematical things “are as separate natures” is therefore contrary to truth, because “it is the *supposition* of their being that way” that is necessary for mathematical knowledge, while “in truth they are derivative” (*Metaphysics* M, 1077a17-20). It is therefore not an *original* separation but a *subsequent* indifference to the dependence on sensible beings that characterizes the manner of being proper to pure numbers. The task of determining how this manner of being itself is to be understood, however, belongs not to mathematics but to “first philosophy (*prôtê philosophia*)” (*Metaphysics* K, 1061b25-27) alone. This is the case because mathematics simply has to “accept (*lambanetai*)” (*Posterior Analytics* A 10, 76a33) the manner of being of the various *original* abstract beings that comprise the *pregiven* contents of arithmetic and geometry, for example, the “one,” the “straight,” the “triangular,” and so on, and deal with them

only insofar as their noncontradictory connections are demonstrable.

It follows for Aristotle from the abstract manner of being proper to the monad that the Platonic solution to the problem of the unity of a number, that is, to the question how the “many” can be understood as “one” at all, is untenable. In the first place, it is untenable because the positing of a “common thing (*koinon*)” above and *alongside* the multitude of units supposedly unified by the integrity of its *genos* attributes unity to something that, properly speaking, cannot be one at all. It cannot be one, because what is meant in speaking of a number is precisely something that is *more* than one thing. Things are one by immediate contact, mixture, or the placement of their parts, none of which are possible when it comes to the monads in the dyad, triad, and so on (*Metaphysics* M, 1082a20-23). Rather, “just as two human beings are not some one thing other than both, so too must it be with the units” (*Metaphysics* M, 1082a23-24). In the second place, Aristotle complains that on account of *what* numbers are one, “no one says anything” (*Metaphysics* L, 1075b34).

Plato’s view of the generic unity of numbers is the consequence of the supposition of the detachment and therefore independence of intelligible monads from sensible beings. This supposition removes the basis for appealing to the natural articulation of ever different and divisible sensible beings to account for the *origin* of the delimitation and unification of single numbers. Having eliminated this ultimate foundation of all possible unity, the *chorismos* thesis seduces the one who posits it into embracing the view that the possibility of collecting together *two* monads in *one* number has to be the effect of an original and therefore independent *genos* or *eidōs*. Thus, for Aristotle, a number is precisely not *one* thing but a

“heap (*sôros*)” (*Metaphysics* H, 1044a4; 1045a8-10; 1084b21-22)⁴ of sensible beings or abstract monads. A number, therefore, is

⁴ Aristotle’s references to a “heap” in connection with the question of the being of number are embedded in discussions that do not explicitly claim that number is like a heap. Rather, in each case number being “like a heap” is presented as the conclusion that follows if number is “not one,” or not “a whole that is something over and above the parts” (*Metaphysics* H, 1044a4-10), or “not some one thing (*hen ti*)” (*Metaphysics* M, 1084b21). Aristotle, however, is clearly intent on establishing that the manner of being of numbers, as a mathematical thing, is “derivative” in the sense that its being does not “take precedence over sensible things” (*Metaphysics* M, 1077a17-20), and, therefore, that “they are not capable of being somewhere as separate” (*Metaphysics* M, 1077b14-15). That from numbers’ incapacity to have being in this regard it follows for Aristotle that “each is not one but is like a heap” (*Metaphysics* H, 1044a4) can be seen as the positive result of his polemic against those people who speak of numbers as being one in the sense that thinghood (*ousia*) is one. Thinghood is not one “in the way they say it is, as though it were a unit or point, but each independent thing (*ousia*) is a complete being-at-work-staying-itself (*entelecheia*), and a particular nature (*phusis*)” (*Metaphysics* H, 1044a7-9); and number for Aristotle is manifestly not one in this sense of an independent thing. Thus, when Aristotle asserts that “it is necessary to a number that there be something by means of which it is one” (*Metaphysics* H, 1044a2-3), he immediately qualifies this assertion (following Ross’ and not Jaeger’s text of the *Metaphysics*) by adding, “that is, if it is one” (*Metaphysics* H, 1044a4); and he goes on to claim that those who make the claim that number is one are unable to say by what means it is so. Aristotle’s point, then, is that those (namely, Plato and other members of the Academy) who claim that number is one are (1) wrong, because their claim is based on the mistaken supposition that number is one as thinghood is one and that therefore the unity of number is capable of being separate from sensible things, (2) deserve criticism because they cannot say by what means number is one, and then (3), because of (1) and (2), number is not some one thing but rather is like a heap.

A syllable is the opposite of a heap according to Aristotle, in that a syllable is something else than the letters (elements) out of which it is composed, the implication clearly being that the heap is not something else than the elements that compose it. In both the case of a heap and a syllable, Aristotle maintains that “the whole is one” (*Metaphysics* Z, 1041b11-12), although the something else than the elements in the instance of a syllable is likened to “the thinghood (*ousia*) of each thing (for that is what is primarily responsible for the being of it),” which is “not an element but a source (*archê*)” of its being an independent thing (*Metaphysics* Z, 1041a31). In the case of the heap, Aristotle’s point is that the whole is not one in this sense. Thus he argues that “if number is separate” and, therefore, “insofar as a number is composite” (*Metaphysics* M, 1084b4), “the one is prior,” but “insofar as the universal and form are prior, the number is; for each of the units is part of the number as its material, but number is the manner of *eidos*.” Because the *eidos* is indivisible, the Platonists say that it is also one, so that both the units, as the parts that compose the *hulê*, and the *eidos* and thinghood (*ousia*) of number are one, and, as such, are sources (*archai*). Aristotle

precisely *nothing more* than these parts, “for a number is only what has been or can be counted” (*Physics* D 14, 223a24-25).

This last point is crucial to Aristotle for understanding properly the soul’s preknowledge of all possible numbers, which, following Plato, could be called a “stored possession (*ktêsis*)” (*Theaetetus*, 197b)—in contrast to a “possession in use (*hexis*)” (*Posterior Analytics* A I, 71a13-28). Because a number is something that coincides with what is counted, the “pure” (that is, “indifferent” to the determinate qualities of sensible beings) intelligible structures available to the soul prior to counting must not be spoken of as *one thing* that, in turn, points to a “common thing (*koinon*)” that should be understood as a whole above and outside of the multitude of counted objects (*Metaphysics* N, 1090a17-18). To the contrary, because the availability of such structures originally becomes known *in* counting, it is likewise rooted in the exercise of counting sensible multitudes and extracting from them, *ex aphairesêôs*, “pure” monads. As a consequence, numbers of “pure” monads involve, no less than numbers of sensible beings, “heaps”—in this case, “heaps” of “pure” monads. They are therefore “one” only in the sense that something can be said to extend “over the whole (*katholou*)” (*Posterior Analytics* B 19, 100a6-8), which rules out their being “one thing” any more than numbers of sensible beings are “one thing” (*Metaphysics* M, 1079a34-36).

holds, however, that “this is impossible”: “for if the number is some one thing and not like a heap,” then its *eidōs* and *hulê* are not only one in different senses, but “in truth each unit has being as a potency (*dunamis*)” and “not as fully at-work (*entelechia*)” (*Metaphysics* M, 1084b22-23). Number, then, cannot be separate as the Platonists say, that is, in accord with the two sources of its being one thing that they identify, the indivisibility of the *eidōs* and the units proper to the *hulê*. Not being separate, it follows that number is not some one thing, but like a heap, that is, like a whole whose being one and therefore *ousia* is derived, in this case, from the multitude of units as such that compose the whole’s elements, in the precise sense of the “how many” indicated by this non-independent whole (see below).

Aristotle's answer to the question that he maintains is unanswered in Plato's generic account of number, namely, *what* it is that is responsible for the unity proper to number, begins by posing it only for *actually* counted multitudes. Such multitudes, as multitudes of homogeneous ones, comprise a unity insofar as each multitude is measured by its own one. Thus Aristotle writes:

We speak of one and many in just the way one might say one and ones, or a white thing and white things, or speak of the things measured off in relation to their measure; in this way, too, manifold things are spoken of, for each number is many because it consists of ones and because each number is measured by the one, and is many as opposed to the one and not to the few. In this sense, then, even two are many, but this not as a multitude having an excess either in relation to anything or simply, but as the first multitude. (*Metaphysics* I, 1056b23-24)

Counting presupposes the homogeneity of that which is counted, which means that in counting one and the same thing is fixed upon, such that its definite amount is arrived at only after *one and the same thing* has been counted over. The "one," then, does *not* have priority in counting as the superiority of a genus over a species, but rather in its character as the "measure (*metron*)" by which the definite amount of a multitude is determined. The one is not a "common (*koinon*) thing" (*Metaphysics* I, 1053a14) over or alongside of things, for "[i]t is clear that the one signifies a measure" (*Metaphysics* N, 1087b33). The "being one" of sensible beings marks both the possibility of their being counted and the *indivisibility* of the "one" that, insofar as it functions to supply the measure of what is counted, is "one sensible thing" and therefore undivided. For example, the "being *one*" of each apple in a number of apples is not divided and therefore does not have a division, even though each apple as a sensible being can be divided, as can any other sensible being. Indivisibility therefore belongs to what is counted only insofar as it is the origin of the measure of the count, because "whatever does not have a division, insofar as it does not

have it, is in that respect called one” (*Metaphysics* D, 1016b4-6). Any specific number is therefore “a multitude measured by the one” (*Metaphysics* I, 1057a3-4). As such, its “thinghood (*ousia*)” is the multitude of units as such, in the precise sense of the “how many” it indicates. Thus thinghood is understood here by Aristotle to be derived, insofar as that *what* each number is, is not something that is separate or detached from the definite amount of homogeneous units it delimits. Thus, for example, “six” units are not “two times three” or “three time two” units, but rather precisely “once six” (*Metaphysics* D, 1020b7-8). For Aristotle, then, there is no such thing as *the six*, with an intelligible being that would be distinct from the many hexads that delimit this or that multitude of “once six” units.

The “indivisible by anything (*pantê adiaireton*)” and “most exact (*akribestaton*)” (*Metaphysics* I, 1053a1) status that the arithmetician understands the “unit (*monas*)” to possess arises for Aristotle on the basis of the elevation of a habitual procedure to the rank of *epistêmê*. The habitual expression of the sensible beings in every count in terms of their “being one”—for example, instead saying “one apple, two apples, three apples,” what is said is rather “one, two, three” (*Metaphysics* M, 1082b35)—points already to the purely arithmetical status of sensible beings as countable material. When this status is abstractedly “lifted off” sensible beings, the mathematical “unit” originates. And it originates as nothing more than the character of being a measure as such, a character expressed through its indivisibility and exactness. The character of the one as measure is what is responsible for the universal applicability of “pure” numbers, namely, of the applicability of the “unit” to any arbitrarily countable being whatsoever. The “unit” is so applicable because its manner of being is *not* separate from the sensible beings that are the source of its abstracted *origin*. Hence, it is only because

sensible beings, as the *kind* of beings that they are, are one and indivisible, that the arithmetician—having already abstractedly posited the “unit” as totally indivisible—is able to then see what always follows from any given sensible being insofar as it is subject to being counted or calculated with as a “unit.” Thus, for example, a human being as the kind of being it is, namely as *human being*, is one and indivisible and, as such, the *abstract* “unit (*monas*)” is applicable to it (*Metaphysics* M, 1078a23-25).

*Aristotle’s Critique and the
Problem of Numbers behind Platonism*

Because for Aristotle it does not follow from the *koinônia* of the units of a number that the number itself is a unity, as it apparently does for Plato and the Platonic view, the only possible “unity” of a number for Aristotle is the unit of the measure employed by the process of counting. The unity of two philosophers, for instance, is therefore “philosopher” for Aristotle, as is the unity of three philosophers, and so on, for however many philosophers one cares to count. The Platonic position that emerges through Aristotle’s polemic against it, to the effect that it is only because there are *eidê* that belong together, and whose *koinônia*, therefore, composes a kinship that yields on the side of their “community” an eidetic number because of the “arithmological” connection of its “members,” addresses precisely the question of the manner of being belonging to the unity proper to different numbers. Thus “the Two itself,” or “the Three itself,” and so on (apparently up to ten—*Metaphysics* L, 1073a20; M, 1084a12-13, 25-26; *Physics* G 6, 206b32-33) can be spoken from the “other” side of the *koinônia* of *eidê* as providing the differentiated unity responsible for the manner of being proper to each different mathematical number. Therefore,

it is only because of the original articulation provided by the *eidê* composing *eidetic numbers* that there can be arbitrarily many mathematical numbers, such as dyads or triads, in both the realm of pure units as well as sensibly perceived things. Whatever the difficulties of such arithmetical Platonism, it seems clear that it addresses a problem posed by numbers that Aristotle did not see, namely, how to account for the differentiated unity proper to each of the basic numbers employed by counting.